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2019 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

LOGIC COLLOQUIUM 2019

Prague, Czech Republic

August 11–16, 2019

Logic Colloquium ’19, the annual European Summer Meeting of the Association of Symbolic Logic, was organized by the Institute of Mathematics of the Czech Academy of Sciences, the Faculty of Arts and Faculty of Mathematics and Physics of the Charles University, and the Faculty of Information Technology of the Czech Technical University in Prague.

The meeting took place from August 11 to August 16, 2019, at the Faculty of Architecture of the Czech Technical University.

The Congress of Logic, Methodology and Philosophy of Science and Technology (CLMPST 2019) was held in Prague in the week before the Logic Colloquium in the same venue. Participants of both conferences were eligible to reduced registration fees.

Major funding for the conference was provided by the Association for Symbolic Logic (ASL), the US National Science Foundation, the Faculty of Arts of the Charles University, and the RSJ foundation.

The success of the meeting was largely due to the excellent work of the Local Organizing Committee under the leadership of its co-chairs, David Chodounský and Jonathan Verner. The other members were Petr Cintula, Radek Honzík, Jan Hubička, Pavel Pudlák, Jan Starý, Šárka Stejskalová, and Neil Thapen.

The Program Committee consisted of Lev Beklemishev (Moscow), Andrew Arana (Paris), Agata Ciabattoni (Vienna), Russell Miller (New York), Martin Otto (Darmstadt), Pavel Pudlák (Prague), Stevo Todorčević (Toronto), and Alex Wilkie (Oxford).

The main topics of the conference were as follows: Computability Theory, Foundations of Geometry, Model Theory, Proof Theory and Proof Complexity, Reflection Principles and Modal Logic, Set Theory. The program included a public evening lecture (a joint event with CLMPST), two tutorial courses, eleven invited lectures, among which the retirement ASL presidential address by Ulrich Kohlenbach, twenty-five invited lectures in six special sessions, and 115 contributed talks. There were 231 participants, and ASL travel grants were awarded to 27 students and recent Ph.Ds.

Hannes Leitgeb (University of Munich) gave the public evening lecture *Ramsification and semantic indeterminacy*.

The following tutorial courses were given:

Michael Rathjen (University of Leeds), *Well-ordering principles in proof theory and reverse mathematics*.

Dilip Raghavan (National University of Singapore), *Higher cardinal invariants*.

The following invited plenary lectures were presented:

Samson Abramsky (Oxford University), *Relating Structure and Power*: *a junction between categorical semantics, model theory and descriptive complexity*.

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Zoé Chatzidakis (Ecole Normale Supérieure), *Notions of difference closures of difference fields*.

Osvaldo Guzman (University of Toronto), *The ultrafilter and almostdisjointness numbers*.

Matthew Harrison-Trainor (Univeristy of Wellington), *Describing countable structures*.

Ulrich Kohlenbach (University of Darmstadt), *Local proof-theoretic foundations, proof-theoretic tameness and proof mining*.

Jan Krajíček (Charles University), *Model theory and proof complexity*.

Vincenzo de Risi (Université Paris-Diderot/CNRS), *Drawing lines through rivers and cities. The meaning of postulates from Euclid to Hilbert*.

Gil Sagi (University of Haifa), *Logic and natural language*: *commitments and constraints*.

Thomas Scanlon (University of California, Berkeley), *Over six decades of the model theory of valued fields*.

Rineke Verbrugge (Groningen University), *Zero-one laws for provability logic and some of its siblings*.

Martin Ziegler (KAIST), *Logic of computing with continuous data*: *Foundations of numerical software engineering*.

More information about the meeting can be found at the conference website, `https://www.lc2019.cz`.

Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
LEV BEKLEMISHEV

Abstract of the Retiring Presidential Address

▶ ULRICH KOHLENBACH, *Local proof-theoretic foundations, proof-theoretic tameness and proof mining*.
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Recently, John Baldwin pointed to a 'paradigm shift in model theory' stressing that while early 20th century logic focused on the formalization of all of mathematics, model theory increasingly studied specific areas of mathematics (local formalizations) with an emphasis on tame structures [1]. We will argue that also the successful use of proof-theoretic methods in core mathematics ('proof mining', [2]) in recent decades was made possible by developing logical metatheorems tailored for applications to particular classes of theorems and proofs in specific areas of mathematics. In analysis, these classes of theorems (e.g., convergence statements), however, do involve arithmetic (together with analytical and geometric structures) and so are not tame in the model-theoretic sense but could in principle display Gödelian or huge growth phenomena. It is an empirical fact, though, that with a few notable exceptions (which still are primitive recursive in the sense of Gödel's T), proofs in existing ordinary analysis are largely tame in the sense of allowing for the extraction of bounds of rather low complexity. To determine the amount of 'proof-theoretic tameness' in a given proof requires a proof-theoretic analysis in each case. We will discuss two recent applications of proof mining, one of which displays a highly tame (polynomial) behavior [3], whereas the other one as it stands uses primitive recursion of type-1 level [4].

[1] J. T. BALDWIN, ***Model Theory and the Philosophy of Mathematical Practice. Formalization without Foundationalism***, Cambridge University Press, 2018.

[2] U. KOHLENBACH, ***Applied Proof Theory*: *Proof Interpretations and their use in Mathematics***, Springer Monographs in Mathematics, Springer Heidelberg-Berlin, 2008.

[3] ———, *A polynomial rate of asymptotic regularity for compositions of projections in Hilbert space*,. ***Foundations of Computational Mathematics***, vol. 19 (2019), pp. 83–99.

[4] U. KOHLENBACH and A. SIPOŞ, *The finitary content of sunny nonexpansive retractions*, `arXiv:1812.04940`, submitted.

Abstracts of Invited Tutorials

▸ DILIP RAGHAVAN, *Higher cardinal invariants*.
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There has been a recent resurgence of research into cardinal invariants at regular uncountable cardinals. This recent work has revealed many differences between cardinal invariants at ω and their analogues at uncountable cardinals. One unexpected conclusion is that there seem to more ZFC inequalities provable at uncountable cardinals than at ω. The study of cardinal invariants at uncountable cardinals has also led to the development of novel forcing techniques, mostly notably the method of Boolean ultrapowers.

I will present a survey of some of this recent work, restricting my attention to certain combinatorial cardinal characteristics at regular uncountable cardinals. Some ZFC results, such as the ones in [1] and [2], as well as some consistency results, such as the ones in [3], will be mentioned. Time permitting, I will expose the method of Boolean ultrapowers as developed in [4] and sketch some of the consistency results at regular uncountable cardinals that can be obtained using this method.

[1] D. Raghavan and S. Shelah, *Two inequalities between cardinal invariants*. ***Fundamenta Mathematicae***, vol. 237 (2017), no. 2, pp. 187–200.

[2] ———, *Two results on cardinal invariants at uncountable cardinals*, ***Proceedings of the 14th and 15th Asian Logic Conferences*** (Mumbai, India and Daejeon, South Korea) (B. Kim, J. Brendle, G. Lee, F. Liu, R. Ramanujam, S. M. Srivastava, A. Tsuboi, and L. Yu, editors), World Scientific Publishing, Hackensack, NJ, 2019, pp. 129–138.

[3] ———, *A small ultrafilter number at smaller cardinals*. ***Archive for Mathematical Logic***, to appear.

[4] ———, *Boolean ultrapowers and iterated forcing*, preprint.

▸ MICHAEL RATHJEN, *Well-ordering principles in proof theory and reverse mathematics*.
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Several results about the equivalence of familiar theories of reverse mathematics with certain well-ordering principles have been proved (Friedman, Marcone, Montalban et al.) by recursion-theoretic and combinatorial methods and also by proof theory (Afshari, Girard, R, Weiermann et al.), employing deduction search trees and cut elimination theorems in infinitary logics with ordinal bounds.

One goal of the talks is to present a general methodology underlying these results which in many cases allows one to establish an equivalence between two types of statements. The first type is concerned with the existence of ω-models of a theory whereas the second type asserts that a certain (usually well-known) elementary operation on orderings preserves the property of being well ordered. These operations are related to ordinal representation systems (ors) that play a central role in proof theory. The question of naturality of ors has vexed logicians for a long time. While ors have a low computational complexity, their "true" nature evades characterization in those terms. One attempt has been to describe their structural properties in category-theoretic terms (Aczel, Feferman, Girard et al.). Some of these ideas will be discussed in the talks.

A second goal is to present rather recent developments (due to Arai, Freund, R), especially work by Freund on higher order well-ordering principles and comprehension.

Abstracts of invited Plenary talks

▸ SAMSON ABRAMSKY, *Relating structure and power*: *a junction between categorical semantics, model theory and descriptive complexity*.
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There is a remarkable divide in the field of logic in Computer Science, between two distinct strands: one focussing on semantics and compositionality ("Structure"), the other on expressiveness and complexity ("Power"). It is remarkable because these two fundamental aspects are studied using almost disjoint technical languages and methods, by almost disjoint research communities. We believe that bridging this divide is a major issue in Computer Science, and may hold the key to fundamental advances in the field.

In this talk, we describe a novel approach to relating categorical semantics, which exemplifies the first strand, to finite model theory, which exemplifies the second. It is based on [1, 2], and ongoing joint work with Nihil Shah, Tom Paine, and Anuj Dawar.

[1] S. Abramsky, A. Dawar, and P. Wang, *The pebbling comonad in finite model theory*, ***Logic in Computer Science (LICS), 2017 32nd Annual ACM/IEEE Symposium***, IEEE, 2017, pp. 1–12.

[2] S. Abramsky and N. Shah, *Relating structure and power: Comonadic semantics for computational resources*, ***27th EACSL Annual Conference on Computer Science Logic, CSL 2018, September 4–7, 2018, Birmingham, UK***, 2018, pp. 2:1–2:17.

► ZOÉ CHATZIDAKIS, *Notions of difference closures of difference fields*.
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It is well known that a differential field K of characteristic 0 is contained in a differential field which is differentially closed and has the property that it K-embeds in every differentially closed field containing K. Such a field is called a differential closure of K, and it is unique up to K-isomorphism. In other words, prime models exist and are unique. The proof uses the fact that the theory of differentially closed fields of characteristic 0 is totally transcendental.

One can ask the same question about difference fields: do they have a difference closure, and is it unique? The immediate answer to both these questions is no, for trivial reasons: in most cases, there are continuum many ways of extending an automorphism of a field to its algebraic closure. Therefore, a natural requirement is to impose that the field K be algebraically closed. Similarly, if the subfield of K fixed by the automorphism is not pseudo-finite, then there are continuum many ways of extending it to a pseudo-finite field, so one needs to add the hypothesis that the fixed subfield of K is pseudo-finite.

In this talk, I will show by an example that even these two conditions do not suffice.

There are two (and more) natural strengthenings of the notion of difference closure, and we show that in characteristic 0, these notions do admit unique prime models over any algebraically closed difference field K, provided the subfield of K fixed by the automorphism is large enough.

In model-theoretic terms, this corresponds to the existence and uniqueness of a-prime or κ-prime models.

In characteristic $p > 0$, no such result can hold.

► VINCENZO DE RISI, *Drawing lines through rivers and cities. The meaning of postulates from Euclid to Hilbert*.
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The talk sketches a history of the development of the meaning of mathematical principles from Antiquity to the Modern Age. Euclid's own conception of principles (definitions, postulates, common notions) was widely different from ours, and it requires some exercise to understand what did it mean for him to ground geometry on a set of principles. We will explore how Euclid's own views on the foundations of mathematics were interpreted and misinterpreted in Late Antiquity, and how a new conception of principles arose in medieval Scholasticism. Such interpretation of axioms and postulates, that stemmed in the commentaries on Aristotle's *Analytics*, was immensely influent in the early modern age, and was endorsed, with various degrees of variance, by authors such as Clavius, Wallis, Leibniz or Euler. In the 18th Century, on the other hand, a new conception of axioms began to arise in the works of Lambert and Bolzano. This last development in the meaning of a mathematical principle paved the way for the modern understanding of it, in the works of Frege, Hilbert, and others. The talk will also present a survey of the main axioms employed in the modern age to ground elementary geometry, which greatly differed from Euclid's original principles and were later collected in the books of the foundations of geometry by Peano, Pasch, and Hilbert.

► OSVALDO GUZMAN, *The ultrafilter and almost disjointness numbers*.
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The *cardinal invariants of the continuum* are certain uncountable cardinals that are less or equal to the cardinality of the real numbers. This relation and nonrelation between this cardinals has been deeply studied by set theorists. In this talk, we will focus on the following two invariants: The *ultrafilter number* $\mathfrak{u}$, which is defined as the smallest size of a base of an ultrafilter, and the *almost disjointness number* $\mathfrak{a}$, which is the smallest size of a MAD family. The consistency of the inequality $\mathfrak{a} < \mathfrak{u}$ is well known and easy to prove. The consistency of the inequality $\mathfrak{u} < \mathfrak{a}$ is much harder to obtain. It was Shelah who proved that, under the assumption that there is a measurable cardinal, there is model of $\omega_1 < \mathfrak{u} < \mathfrak{a}$. In spite of the beauty of the result, the following questions remained open:

(Shelah) Does $CON(ZFC)$ implies $CON(ZFC + \mathfrak{u} < \mathfrak{a})$?

(Brendle) Is it consistent that $\omega_1 = \mathfrak{u} < \mathfrak{a}$?

Acknowledgments. In this talk, we are going to see how to provide a positive answer to both questions. This is joint work with Damjan Kalajdzievski. No previous knowledge of cardinal invariants of the continuum is needed for the talk.

► MATTHEW HARRISON-TRAINOR, *Describing countable structures*.
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Given a countable structure, how do we measure its complexity? One way to do this is by measuring the complexity of describing that structure. Dana Scott proved that for each countable structure $\mathcal{A}$, there is a sentence of infinitary logic that is true of $\mathcal{A}$ and not true of any other countable structure. We can think of such a sentence as a description of the structure, and call any such sentence a Scott sentence. The Scott complexity of a structure is the complexity of the simplest Scott sentence for that structure. The Scott complexity of a structure is tightly related to other notions of complexity, such as the complexity of understanding automorphisms of the structure, or of finding isomorphisms between different copies of the structure. This talk will begin with a general overview of the area followed by a number of recent results on finitely generated structures and on structures of high Scott rank.

► JAN KRAJÍČEK, *Model theory and proof complexity*.
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Mathematical logic and computational complexity theory have many topics in common. In most cases the links between the two fields are fostered by finite combinatorics, manifesting either via proof theory or via finite model theory.

There are, however, also topics in complexity theory where infinitary methods of logic shed a new light on old problems. I will discuss, in particular, how nonfinite model theory relates to proof complexity. The relevant model theoretical problems involve constructions of models of bounded arithmetic and of expanded extensions of pseudo-finite structures. I will describe forcing with random variables aimed at tackling these problems, and give some examples of results that can be obtained in this way.

► HANNES LEITGEB, *Ramsification and semantic indeterminacy*.
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Since the publication of Ramsey's (1929) Theories, the Ramsification of scientific theories has become a major tool in theory interpretation and reconstruction. In this talk, I will argue that the Ramsification of classical semantics can also help us overcome problems that result from the vagueness of ordinary terms in natural language or from the theoreticity and open-endedness of technical terms in mathematical and scientific language. The resulting "Ramsey semantics" saves all of classical logic and almost all of classical semantics, while embracing semantic indeterminacy without going down an epistemicist or supervaluationist road.

▸ GIL SAGI, *Logic and natural language*: *commitments and constraints*.
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Most of the contemporary research in logic is carried out with respect to formal languages. Logic, however, is said to be concerned with correct reasoning, and it is natural language that we usually reason in. Thus, in order to assess the validity of arguments in natural language, it is useful to formalize them: to provide matching arguments in a formal language where logical properties become perspicuous. It has been recognized in the literature that formalization is far from a trivial process. One must discern the logical from the nonlogical in the sentence, a process that requires theorizing that goes beyond the mere understanding of the sentence formalized [1]. Moreover, according to some, "logical forms are not to be discovered but rather established and ascribed to expressions within processes of the reflective equilibrium" [2]. I concur. I argue that logical forms are imposed, and that furthermore, they carry a normative force in the form of commitments on behalf of the theorizer.

In previous work [3], I proposed a model-theoretic framework of "semantic constraints", where there is no strict distinction between logical and nonlogical vocabulary. The form of sentences in a formal language is determined rather by a set of constraints on models. In the present article, I show how this framework can also be used in the process of formalization, where the semantic constraints are conceived of as commitments made with respect to the language.

[1] G. Brun, *Reconstructing arguments*: *Formalization and reflective equilibrium*. ***Logical Analysis and History of Philosophy***, vol. 17 (2014), pp. 94–129.

[2] J. Peregrin and V. Svoboda, ***Reflective Equilibrium and the Principles of Logical Analysis*: *Understanding the Laws of Logic***, Routledge Studies in Contemporary Philosophy, Routledge, 2017.

[3] G. Sagi, *Formality in logic*: *From logical terms to semantic constraints*. ***Logique et Analyse***, vol. 227 (2014), pp. 259–276.

▸ THOMAS SCANLON, *Over six decades of the model theory of valuedfields*.
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Inspired by Angus Macintyre's lecture "Twenty years of p-adic model theory" at the Logic Colloquium '84 in Manchester, I widen the scope exploring the role that the theory of valued fields has played (and continues to play) in the internal development of model theory and in the applications of model theory to other parts of mathematics.

▸ RINEKE VERBRUGGE, *Zero-one laws for provability logic and some of its siblings*.
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Glebskii and colleagues proved in the late 1960s that each formula of first-order logic without constants and function symbols obeys a zero-one law. That is, every such formula is either almost surely valid or almost surely not valid: As the number of elements of finite models increases, each formula holds either in almost all or in almost no models of that size. As a consequence, many properties of models, such as having an even number of elements, cannot be expressed in the language of first-order logic without constants and function symbols. In a 1994 article, Halpern and Kapron proved similar zero-one laws for classes of models corresponding to the modal logics K, T, S4, and S5.

In this presentation, we discuss zero-one laws for some modal logics that impose structural restrictions on their models; all three logics that we are interested in are sound and complete with respect to finite partial orders, with different extra restrictions per logic. We prove zero-one laws for provability logic and its two siblings Grzegorczyk logic and weak Grzegorczyk logic, with respect to model validity. Moreover, for all three logics, we axiomatize validity in almost all relevant finite models, leading to three different axiom systems. In the proofs, we use a combinatorial result by Kleitman and Rothschild about the structure of almost all finite partial orders. We also discuss the question whether for the three sibling logics, validity in almost all relevant finite frames can be axiomatized as well. Finally, we consider the complexity of deciding whether a given formula is almost surely valid in the relevant finite models.

▸ MARTIN ZIEGLER, *Logic of Computing with Continuous Data*: *Foundations of Numerical Software Engineering*.
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Over 30 years after introducing the IEEE 754 standard, Numerics still gyrates around floating point numbers: from specification (e.g., of `e04bbc` in the NAG library) via analysis (unit-cost/realRAM/Blum–Shub–Smale model) and implementation to verification. Yet their violation of Distributive Law, of Intermediate-Value Theorem, and of Quantifier Elimination hampers rigorous approaches to Numerical Software Engineering: Modern Calculus builds on real (rather than rational) numbers for a reason!

We reconcile the convenient algebraic perspective on real computation (Bürgisser) with Computable Analysis (Grzegorczyk, Pour-El, Weihrauch) by developing Turing-complete semantics for operating on continuous structures (Poizat, Zucker). This imperative counterpart to realPCF (Escardo) extends the powerful formal tools of Software Engineering from the discrete to the continuous realm with benefits to numerical practice.

Abstracts of invited talks in the Special Session on Computability

▸ LAURENT BIENVENU, BARBARA F. CSIMA, AND MATTHEW HARRISON-TRAINOR, *Some questions of uniformity in algorithmic randomness*.
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The Ω numbers—the halting probabilities of universal prefix-free machines—are known to be exactly the Martin-Löf random left-c.e. reals [3, 4, 5]. It was previously open however whether this equivalence was uniform, that is, whether one can uniformly produce, from a Martin-Löf random left-c.e. real α, a universal machine U whose halting probability is α (see e.g., [1]). We answer this question in the negative. We also answer a question of Barmpalias and Lewis-Pye [2] by showing that given a left-c.e. real α, one cannot uniformly produce a left-c.e. real β such that $\alpha - \beta$ is neither left-c.e. nor right-c.e.

[1] G. Barmpalias, *Aspects of Chaitin's Omega*, ***Algorithmic Randomness: Progress and Prospects*** (J. Franklin and C. Porter, editors), Springer, 2018, pp. 623–632.

[2] G. Barmpalias and A. Lewis-Pye, *A note on the differences of computably enumerable reals*, ***Computability and Complexity***, Lecture Notes in Computer Science, vol. 10010, Springer, 2017, pp. 623–632.

[3] C. S. Calude, P. H. Hertling, B. Khoussainov, and Y. Wang, *Recursively enumerable reals and Chaitin Ω numbers*. ***Theoretical Computer Science***, vol. 255 (2001), no. 1–2, pp. 125–149.

[4] G. J. Chaitin, *A theory of program size formally identical to information theory*. ***Journal of the ACM***, vol. 22 (1975), pp. 329–340.

[5] A. Kučera and T. Slaman, *Randomness and recursive enumerability*. ***SIAM Journal on Computing***, vol. 31 (2001), pp. 199–211.

▸ WESLEY CALVERT, DOUGLAS CENZER, AND VALENTINA HARIZANOV, *Approximately computable equivalence structures*.
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In the past, we investigated computable, computably enumerable, and co-computably enumerable equivalence structures and their isomorphisms [2, 3]. In recent years, various authors investigated approximate computability for sets and reducibilities. We introduce and study the notions of generic and coarse computability for equivalence structures and their isomorphisms [1]. A binary relation R on ω is *generically computable* if there is a partial computable function $\varphi\colon \omega^2 \to \{0,1\}$ such that on its domain, φ coincides with the characteristic function of R and, furthermore, φ is defined on $A \times A$ for a computably enumerable set A of asymptotic density 1. A set $B \subseteq \omega$ is called R-*faithful* if, whenever aRb, then $a \in B$ iff $b \in B$. We say that a generically computable R is faithfully generically computable if the corresponding set A is R-faithful. We show that every equivalence structure has a generically computable copy. We also show that an equivalence structure $\mathcal{E}$ has a faithfully generically computable copy if and only if $\mathcal{E}$ has an infinite faithful substructure with a computable copy.

An equivalence structure $\mathcal{E} = (\omega, E)$ is *coarsely computable* if there is a computable equivalence relation C such that E and C agree on a set $A \subseteq \omega$ of asymptotic density 1. The structure $\mathcal{E}$ is faithfully coarsely computable if A is both C-faithful and E-faithful. Every equivalence structure has a coarsely computable copy. Not every faithfully coarsely computable equivalence structure has a faithfully generically computable copy, and not every equivalence structure has a faithfully coarsely computable copy. We also investigate generically and coarsely computable isomorphisms and how their categoricity differs from computable categoricity.

[1] W. Calvert, D. Cenzer, and V. Harizanov, *Generically computable equivalence structures and isomorphisms*, https://arxiv.org/abs/1808.02782.

[2] W. Calvert, D. Cenzer, V. Harizanov, and A. Morozov, *Effective categoricity of equivalence structures*. ***Annals of Pure and Applied Logic***, vol. 141 (2006), pp. 61–78.

[3] D. Cenzer, V. Harizanov, and J. B. Remmel, Σ^0_1 *and* Π^0_1 *equivalence structures*. ***Annals of Pure and Applied Logic***, vol. 162 (2011), pp. 490–503.

▸ DENIS HIRSCHFELDT, *Computability theory, reverse mathematics, and Hindman's Theorem*.
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I will discuss results and open problems concerning the computability-theoretic and reverse-mathematical strength of versions of Hindman's Theorem, which states that for any coloring of the natural numbers with finitely many colors, there is an infinite set S such that all nonempty sums of distinct elements of S have the same color.

▸ NOAH SCHWEBER, *More effective cardinal characteristics*.
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A *cardinal characteristic of the continuum* is a measure of the difficulty of finding a "sufficiently large" set of reals for a given task—for example, the smallest cardinality of a set of functions from naturals to naturals such that every function is dominated by one in the set, or the smallest cardinality of a nonmeasurable set. While these are purely set-theoretic objects, they often have computability-theoretic analogues—degree notions which similarly measure the difficulty of creating sufficient sets, but this time from a computational perspective.

In this talk, I'll present work, joint with Ivan Ongay-Valverde, on a new class of effective cardinal characteristics. They form the effective analogue of problems such as "How large does a set of 2-branching subtrees of $3^{<\omega}$ have to be in order for every element of 3^{ω} to be a path through one of the trees?" We will show that on the effective side we get multiple distinct hierarchies, and discuss their interactions with classical computability-theoretic notions such as computable traceability.

Time permitting, I'll also say a bit about another less-studied appearance of cardinal characteristics in computability theory—this time, in computable structure theory (this part joint with Uri Andrews, Joe Miller, and Mariya Soskova).

Abstracts of invited talks in the Special Session on Foundations of Geometry

► MICHAEL BEESON, *On the notion of equal figures in Euclid*.
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Euclid uses an undefined notion of "equal figures", to which he applies the common notions about equals added to equals or subtracted from equals. When we formalized Euclid Book I for computer proof-checking, we had to add fifteen axioms about undefined relations "equal triangles" and "equal quadrilaterals" to replace Euclid's use of the common notions. In this article, we offer definitions of "equal triangles" and "equal quadrilaterals, that Euclid could have given, and prove that they have the required properties, by proofs Euclid could have given. This removes the need for adding new axioms.

► PIERRE BOUTRY, *Towards an independent version of Tarski's system of geometry*.
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In 1926–1927, Tarski designed a set of axioms for Euclidean geometry which reached its final form in a manuscript by Schwabhäuser, Szmielew and Tarski in 1983. The differences amount to simplifications obtained by Tarski and Gupta. Gupta presented an independent version of Tarski's system of geometry, thus establishing that his version could not be further simplified without modifying the axioms. To obtain the independence of one of his axioms, namely Pasch's axiom, he proved the independence of one of its consequence: the previously eliminated symmetry of betweenness. However, an independence model for the nondegenerate part of Pasch's axiom was provided by Szczerba for another version of Tarski's system of geometry in which the symmetry of betweenness holds. This independence proof cannot be directly used for Gupta's version as the statements of the parallel postulate differ.

In this talk, we present our progress towards obtaining an independent version of a variant of Gupta's system. Compared to Gupta's version, we split Pasch's axiom into this previously eliminated axiom and its nondegenerate part and change the statement of the parallel postulate. To select this statement, our previous article, *Parallel postulates and continuity axioms: a mechanized study in intuitionistic logicusing Coq*, proved to be useful so we detail some of these results.

► JOHN MUMMA, *Diagrams and parallelism*.
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The topic of my talk is how the relation of parallelism is represented in diagrammatic proofs of plane elementary geometry. I will discuss how the boundedness of diagrams motivates a constructive conception of the relation, and consider how the formal system presented in [1] can be modified in accord with this conception.

[1] J. Avigad, E. Dean, and J. Mumma, *A formal system for Euclid's elements*. ***The Review of Symbolic Logic***, vol. 2 (2009), no. 4, pp. 700–768.

▸ GIANLUCA PAOLINI, *First-order model theory of free projective planes*.
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We prove that the theory of open projective planes is complete and strictly stable, and infer from this that Marshall Hall's free projective planes ($\pi^n : 4 \leq n \leq \omega$) are all elementary equivalent and that their common theory is strictly stable and decidable, being in fact the theory of open projective planes. We further characterize the elementary substructure relation in the class of open projective planes, and show that ($\pi^n : 4 \leq n \leq \omega$) is an elementary chain. We then prove that for every infinite cardinality κ there are 2^κ nonisomorphic open projective planes of power κ, improving known results on the number of open projective planes. Finally, we characterize the forking independence relation in models of the theory and prove that π^ω is strongly type-homogeneous.

[1] G. Paolini and T. Hyttinen, *First-order model theory of free projective planes*: *Part I*, submitted.

Abstracts of invited talks in the Special Session on Model Theory

▸ AYŞE BERKMAN, *Sharp actions of groups in the finite Morley rank context*.
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After introducing basics on permutation groups of finite Morley rank, I plan to focus on sharply 2-transitive and generically sharply n-transitive group actions in the finite Morley rank setting.

Let G be a group acting on a set X and fix a positive integer n. If for any two n-tuples $(x_1, \dots, x_n)$ and $(y_1, \dots, y_n)$ consisting of distinct elements of X, there exists a (unique) $g \in G$ such that $gx_i = y_i$ for all $i = 1, \dots, n$, then we say G acts (sharply) n-transitively on X.

For any field (or more generally, for any near-field) K, the action of the group of affine K-linear transformations on K viewed as an affine line, that is $K^* \ltimes K^+ \curvearrowright K$, is sharply 2-transitive. We call such actions standard sharply 2-transitive actions. Sharply 2-transitive finite groups were classified by Zassenhaus in 1936. For a long time, it had been an open question whether every infinite sharply 2-transitive group is standard or not. Finally in 2017, Rips, Segev, and Tent, in 2016, Tent and Ziegler; constructed examples of sharply 2-transitive groups which are not standard. However, their examples are not of finite Morley rank. Hence the problem remains open in the finite Morley rank context.

In my talk, first I shall talk about the following partial solution to the problem.

Theorem 1 (Altınel, B., Wagner, 2019). *Let G be an infinite sharply 2-transitive group of finite Morley rank, and of characteristic p. Then the following holds.*
(a) *If $p = 3$, then G is standard.*
(b) *If $p = 2$, then G splits.*
(c) *If $p \neq 2$ and G splits, then G is standard.*

In a sharply 2-transitive group, if the stabilizer of an element has no involutions, then we say that the characteristic of the group is 2. Otherwise, all strongly real elements (i.e., products of two distinct involutions) are conjugate, and their orders are equal to some prime $p \geqslant 3$, or they are of infinite order. In this case, we say the characteristic of the group is p or

0, respectively. If $G = N \rtimes \mathrm{stab}(x)$ for some $x \in X$ and normal subgroup $N \trianglelefteq G$, then we say G splits.

The second part of my talk will be devoted to the study of generically sharply n-transitive groups. More precisely, I shall talk about the following theorem.

THEOREM 2 (B., Borovik, 2018). *Let G be a group of finite Morley rank, and V a connected abelian group of Morley rank n with no involutions. Assume that G acts definably and generically sharply n-transitively on V, then there is an algebraically closed field F of characteristic not 2, such that $G \curvearrowright V$ is equivalent to $\mathrm{GL}_n(F) \curvearrowright F^n$.*

If G is sharply transitive on a generic subset of X, then we say G acts generically sharply transitively on X. Similarly, if the induced action of G on X^n is generically sharply transitive, then we say G acts generically sharply n-transitively on X.

▸ PHILIP DITTMANN, *Models of the common theory of algebraic extensions of the rational numbers*.
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Although the theory of algebraic extensions of $\mathbb{Q}$ has many properties normally seen as undesirable(for instance, it is not computably enumerable and has many completions with bad stability properties), it still makes sense to investigate its nonstandard models. Using the model theory of local fields, as well as some algebraic ingredients interesting in their own right, one can show that every such "nonstandard algebraic" field is dense in all its real and p-adic closures. Along the way, we will encounter the classical notion of the Pythagoras number from field theory, as well as a new p-adic version of the same, inspired by axiomatisations of the universal theory of local fields. As a consequence of the denseness, we obtain a result on definability of the valuation ring in henselian fields whose residue field is a number field.

Acknowledgment. This is joint work with Sylvy Anscombe and Arno Fehm.

▸ ANGUS MACINTYRE, *Model theory of adeles. Arithmetic equivalence*.
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Let A_K be the ring of adeles of a number field K. Only after Ax had given his analyses of uniform definability and decidability for the completions of K at the its standard absolute values (fifty years ago) could one give informative analyses of the definability and decidability for the individual A_K. This was first done, early on, by Weisspfenning. Much later, Derakhshan and I have given a more algebraic treatment purely in the language of rings. Still, many questions remain unanswered, notably that of definability and decidability uniformly in K. This is related to basic issues of unbounded ramification (going back to Herbrand's work in algebraic number theory). Some of these issues will be sketched, but the main emphasis will be on a question posed in other terms by number theorists more than eighty years ago. The question asks to what extent A_K determines K. It has been known for a long time that A_K does not determine K (up to isomorphism) in general, and much fine structure has been discovered (involving Galois theory, zeta functions, class numbers, etc). In the talk I will give a thorough analysis of elementary equivalence for adele rings, and show that it coincides with isomorphism. I also reformulate some work of the number theorists to show that for any K there are at most finitely many L so that A_K and A_L are isomorphic.

Acknowledgment. This work is joint with J. Derakhshan (Oxford).

▸ FRANCESCO PARENTE, *Model-theoretic properties of ultrafilters and universality of forcing extensions*.
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In this talk, I will discuss some recent results at the interface between model theory and set theory. The first part will be concerned with model-theoretic properties of ultrafilters in the context of Keisler's order. I will use the framework of 'separation of variables', recently developed by Malliaris and Shelah, to provide a new characterization of Keisler's order in terms of saturation of Boolean ultrapowers. Furthermore, I will show that good ultrafilters on complete Boolean algebras are precisely the ones which capture the maximum class in Keisler's order, answering a question posed by Benda in 1974.

In the second part of the talk, I will report on joint work with Matteo Viale in which we apply the above results to the study of models of set theory. In particular, our work aims at understanding the universality properties of forcing extensions. To this end, we analyse Boolean ultrapowers of H_{ω_1} in the presence of large cardinals and give a new interpretation of Woodin's absoluteness results in this context.

Abstracts of invited talks in the Special Session on Proof Theory and Proof Complexity

▸ BAHAREH AFSHARI, *An infinitary treatment of fixed point modal logic*.
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Fixed point modal logic deals with the concepts of induction and recursion in a most fundamental way. The term refers to any logic built on the foundation of modal logic that features inductively and/or co-inductively defined operators. Examples range from simple temporal logics (e.g., tense logic and linear time logic) to the highly expressive modal μ-calculus and its extensions.

We explore the proof theory of fixed point modal logic with converse modalities, commonly known as 'full μ-calculus'. Building on nested sequent calculi for tense logics [2] and infinitary proof theory of fixed point logics [1], a cut-free sound and complete proof system for full μ-calculus is proposed. As a result of the framework, we obtain a direct proof of the regular model property for the logic (originally proved in [4]): every satisfiable formula has a tree model with finitely many distinct subtrees (up to isomorphism). Many of the results appeal to the basic theory of well-quasi-orders in the spirit of Kozen's proof of the finite model property for μ-calculus [3].

This talk is based on joint work with Gerhard Jäger (University of Bern) and Graham E. Leigh (University of Gothenburg).

[1] G. Jäger, M. Kretz, and T. Studer, *Canonical completeness of infinitary mu*. ***The Journal of Logic and Algebraic Programming***, vol. 76.2 (2008), pp. 270–292.

[2] R. Kashima, *Cut-free sequent calculi for some tense logics*. ***Studia Logica***, vol. 53.1 (1994), pp. 119–135.

[3] D. Kozen, *Cut-free sequent calculi for some tense logics*. ***Theoretical Computer Science***, vol. 27 (1983), pp. 333–354.

[4] M. Vardi, *Reasoning about the past with two-way automata*, ***Automata, Languages and Programming*** (K. G. Larsen, S. Skyum, and G. Winskel, editors), Springer, Berlin Heidelberg, Warsaw, Poland, 1998, pp. 628–641.

▸ OLAF BEYERSDORFF, *Proof complexity of quantified Boolean formulas*.
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Proof complexity of quantified Boolean formulas (QBF) studies different formal calculi for proving QBFs and compares them with respect to the size of proofs. There exists a number of conceptually quite different QBF resolution calculi, modelling QBF solving approaches, as well as QBF cutting planes, algebraic systems, Frege systems, and sequent calculi. We give an overview of the relative proof complexity landscape of these systems.

From a complexity perspective it is particularly interesting to understand which lower bound techniques are applicable in QBF proof complexity. While some propositional techniques, such as feasible interpolation [3] and game-theoretic approaches [4], can be lifted to QBF, QBF proof complexity also offers completely different approaches that do not have analogues in the propositional domain. These build on strategy extraction, whereby from a refutation of a false QBF a countermodel can be efficiently constructed. Extracting strategies in restricted computational models (such as bounded-depth circuits) and exhibiting false QBFs where countermodels are hard to compute in the same computational model leads to lower bounds for the size of proofs in QBF calculi.

We explain this paradigm for prominent QBFs [2, 1]. For QBF Frege systems this approach even characterises QBF Frege lower bounds by circuit lower bounds [5]. This provides a strong link between circuit complexity and QBF proof complexity, unparalleled in propositional proof complexity.

This line of research also intrinsically connects to QBF solving as different QBF resolution calculi form the basis for different approaches in QBF solving such as QCDCL [7] and QBF expansion [6]. Thus QBF proof complexity provides the main theoretical tool towards an understanding of the relative power and limitations of these powerful algorithms.

[1] O. Beyersdorff, J. Blinkhorn, and L. Hinde, *Size, cost, and capacity: A semantic technique for hard random QBFs*, ***Proceedings of the Conference on Innovations in Theoretical Computer Science (ITCS)***, 2018, pp. 9:1–9:18.

[2] O. Beyersdorff, I. Bonacina, and L. Chew, *Lower bounds: From circuits to QBF proof systems*, ***Proceedings of the ACM Conference on Innovations in Theoretical Computer Science (ITCS)***, ACM, 2016, pp. 249–260.

[3] O. Beyersdorff, L. Chew, M. Mahajan, and A. Shukla, *Feasible interpolation for QBF resolution calculi*. ***Logical Methods in Computer Science***, vol. 13 (2017).

[4] O. Beyersdorff, L. Chew, and K. Sreenivasaiah, *A game characterisation of tree-like Q-resolution size*. ***Journal of Computer and System Sciences***, (2017), in press.

[5] O. Beyersdorff and J. Pich, *Understanding Gentzen and Frege systems for QBF*, ***Proceedings of the ACM/IEEE Symposium on Logic in Computer Science (LICS)***, 2016.

[6] M. Janota and J. Marques-Silva, *Expansion-based QBF solving versus Q-resolution*. ***Theoretical Computer Science***, vol. 577 (2015), pp. 25–42.

[7] L. Zhang and S. Malik, *Conflict driven learning in a quantified boolean satisfiability solver*, ***IEEE/ACM International Conference on Computer-Aided Design (ICCAD)***, 2002, pp. 442–449.

► SARA NEGRI, *Syntax for semantics*.
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A general method is presented for converting semantics into well-behaved proof systems. Previous work has shown that the method works in full generality for Kripke semantics. A number of extensions thereof, covering preferential and neighbourhood semantics, will be surveyed to highlight its uniform features.

► PEDRO PINTO, *Proof mining with the bounded functional interpretation*.
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In the context of the proof mining research program [5, 6], the standard tool guiding the extraction of new information from noneffective mathematical proofs is Ulrich Kohlenbach's monotone functional interpretation. In 2005, a different interpretation was introduced by Fernando Ferreira and Paulo Oliva, the bounded functional interpretation [4]. We will look at some of the first applications of this functional interpretation to the proof mining of concrete results. In [3], we explained how certain sequential weak compactness arguments can be eliminated from proof mining and used this ideia to obtain a quantitative version of Bauschke's theorem from [1]. Bounds on the metastability (in the sense of Terence Tao) for variants of the proximal point algorithm were obtained in [7, 8, 2].

Acknowledgment. This is partly joint work with Bruno Dinis, Fernando Ferreira, and Laurenţiu Leuştean.

[1] H. H. Bauschke, *The approximation of fixed points of compositions of nonexpansive mappings in Hilbert space*. ***Journal of Mathematical Analysis and Applications***, vol. 202 (1996), no. 1, pp. 150–159.

[2] B. Dinis and P. Pinto, *Metastability of the proximal point algorithm with multi-parameters*, in preparation.

[3] F. Ferreira, L. Leuştean, and P. Pinto, *On the removal of weak compactness arguments in proof mining*, submitted, preprint, `arXiv:1810.01508`.

[4] F. Ferreira and P. Oliva, *Bounded functional interpretation*. ***Annals of Pure and Applied Logic***, vol. 135 (2005), no. 1–3, pp. 73–112.

[5] U. Kohlenbach, ***Applied Proof Theory: Proof Interpretations and their use in Mathematics***, Springer Monographs in Mathematics, Springer-Verlag, 2008.

[6] ———, *Proof-theoretic methods in nonlinear analysis*, ***Proceedings of the International Congress of Mathematicians 2018, vol. 2*** (B. Sirakov, P. Ney de Souza, and M. Viana, editors), World Scientific, Rio de Janeiro, Brazil, 2019, pp. 61–82.

[7] L. Leuştean and P. Pinto, *Quantitative results on Halpern type proximal point algorithms*, in preparation.

[8] P. Pinto, *Quantitative version of a theorem by H-K. Xu*, in preparation.

► THOMAS POWELL, *A new application of proof mining in the fixed point theory of uniformly convex Banach spaces*.
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Proof mining is a branch of mathematical logic which makes use of proof theoretic techniques to extract quantitative information from seemingly nonconstructive proofs. In this talk, I present a new application of proof mining in functional analysis, which focuses on the convergence of the Picard iterates $(T^n x)_{n\in\mathbb{N}}$ for a class of mappings T on uniformly convex Banach spaces whose fixpoint sets have nonempty interior.

► NEIL THAPEN, *Induction, search problems and approximate counting*.
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An important open problem in bounded arithmetic is to show that (in the presence of an oracle predicate) theories with more induction are strictly stronger when it comes to proving sentences of some fixed complexity. In classical fragments of Peano arithmetic, the Π_1 consequences of theories can be separated by consistency statements, and the Π_2 consequences by the growth-rate of definable functions. In bounded arithmetic, neither of these seems to be possible.

I will discuss this problem, and describe some recent progress on it. A particular instance of the problem is to find a $\forall\Sigma^b_1$ sentence which is provable in full bounded arithmetic but not in T^2_2 (i.e., with induction restricted to Σ^b_2 formulas). In [1] we study the theory APC_2, which allows approximate counting of Σ^b_1 sets, and appears to have a broadly similar level of strength to T^2_2. We find such a $\forall\Sigma^b_1$ sentence separating APC_2 from full bounded arithmetic, using a probabilistic oracle construction based on a simplified switching lemma.

[1] L. A. Kołodziejczyk and N. Thapen, *Approximate counting and NP search problems*, preprint, `arXiv:1812.10771`.

Abstracts of invited talks in the Special Session on Reflection Principles and Modal Logic

► ALI ENAYAT, *Some recent news about truth theories*.
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For a fragment B of PA (Peano arithmetic), $\mathsf{CT}^-[\mathsf{B}]$ (compositional truth over B) is the theory formulated in the language of arithmetic augmented with a fresh predicate $\mathsf{T}(x)$ to express: "x is the Gödel number of a true arithmetical sentence". The axioms of $\mathsf{CT}^-[\mathsf{B}]$ consist of the axioms of B plus finitely many sentences that stipulate that $\mathsf{T}(x)$ is well behaved on atomic sentences, and obeys Tarski's familiar compositional clauses guiding the behaviour of the truth predicate. We have known, since the pioneering work of Krajewski, Kotlarski, and Lachlan (1981), that $\mathsf{CT}^-[\mathsf{PA}]$ is conservative over PA. In this talk, we will discuss the following recent developments:

- Recent joint work [1] of Pakhomov and the author on the equivalence of $\mathsf{CT}^-[\mathsf{I}\Delta_0 + \mathsf{Exp}] + \mathsf{DC}$ with $\mathsf{CT}_0[\mathsf{PA}]$, where DC is the axiom stating "a disjunction of finitely many sentences is true iff one of the disjuncts is true"; and $\mathsf{CT}_0[\mathsf{PA}]$ is the result of adding the induction scheme for Δ_0-formulae that mention the truth predicate to $\mathsf{CT}^-[\mathsf{PA}]$. This result refines earlier work by Kotlarski (1986) and Cieśliński (2010) and shows that $\mathsf{CT}^-[\mathsf{PA}] + \mathsf{DC}$ is not conservative over PA, since as demonstrated by Wcisło and Łełyk [3], $\mathsf{CT}_0[\mathsf{PA}]$ proves $\mathsf{Con}(\mathsf{PA})$ (and much more).
- Recent joint work [2] of Łełyk, Wcisło, and the author on the *feasible reducibility* of $\mathsf{CT}^-[\mathsf{PA}]$, and certain other canonical untyped truth theories to PA. In particular, this shows that $\mathsf{CT}^-[\mathsf{PA}]$ does not exhibit superpolynomial speed-up over PA, in sharp contrast to the superexponential speed-up of $\mathsf{CT}^-[\mathsf{B}]$ over B for finitely axiomatizable B.

[1] A. Enayat and F. Pakhomov, *Truth, disjunction, and induction*. ***Archive for Mathematical Logic***, 2019, `https://doi.org/10.1007/s00153-018-0657-9`.

[2] A. Enayat, M. Łełyk, and B. Wcisło, *Truth and feasible reducibility*. ***The Journal of Symbolic Logic***, to appear, 2019, `arXiv:1902.00392`.

[3] B. Wcisło and M. Łełyk, *Notes on bounded induction for the compositional truth predicate*. ***The Review of Symbolic Logic***, vol. 10 (2017), pp. 455–480.

▶ EMIL JEŘÁBEK, *Reflection principles in weak and strong arithmetics*.
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Reflection principles are established as an important tool in the study of first-order theories of arithmetic. In the realm of strong fragments of arithmetic (say, above $I\Delta_0 + EXP$), this means *first-order reflection principles* expressing the soundness of subsystems of arithmetic itself with respect to formulas of bounded complexity. First-order reflection schemata come in various shapes depending on their purpose (uniform reflection principles, local reflection principles, reflection rules), and since they operate inside with the same language as outside, they can be iterated.

This approach is of no use for weak theories of arithmetic such as fragments of bounded arithmetic $I\Delta_0 + \Omega_1$, since these theories cannot even prove the consistency of the base theory Q. However, fragments of bounded arithmetic can be analyzed using reflection principles for *propositional proof systems*, expressing that tautologies of bounded complexity provable in the system are true under Boolean assignments. Using translation of bounded formulas into propositional language, these reflection principles can be themselves expressed by sequences of propositional tautologies.

In this talk, I will review basic properties of reflection principles in both setups, highlighting what makes them similar and what makes them different.

▶ FEDOR PAKHOMOV, *A weak set theory that proves its own consistency*.
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We introduce a weak set theory $\mathsf{H}_{<\omega}$. A formalization of arithmetic on finite von Neumann ordinals gives an embedding of arithmetical language into this theory. We show that $\mathsf{H}_{<\omega}$ proves a natural arithmetization of its own Hilbert-style consistency. Unlike the previous examples (due to Willard [2]) of theories proving their own consistency, $\mathsf{H}_{<\omega}$ appears to be sufficiently natural.

The theory $\mathsf{H}_{<\omega}$ is infinitely axiomatizable and proves existence of all individual hereditarily finite sets, but at the same time all its finite subtheories have finite models. Therefore, our example avoids the strong version of Gödel's second incompleteness theorem (due to Pudlák) that asserts that no consistent theory interpreting Robinson's arithmetic Q proves its own consistency [1]. To show that $\mathsf{H}_{<\omega}$ proves its own consistency we establish a conservation result connecting Kalmar elementary arithmetic EA and $\mathsf{H}_{<\omega}$.

The theory $\mathsf{H}_{<\omega}$ is a first-order theory in the signature with equality $=$, membership predicate $\in$, and unary function $\overline{\mathsf{V}}$. Axioms of $\mathsf{H}_{<\omega}$:

1. $x = y \leftrightarrow \forall z(z \in x \leftrightarrow z \in y)$ (Extensionality);
2. $\exists y \forall z(z \in y \leftrightarrow z \in x \wedge \varphi(z))$ (Separation);
3. $y \in \overline{\mathsf{V}}(x) \leftrightarrow (\exists z \in x)(y \subseteq \overline{\mathsf{V}}(z))$ (Defining axiom for $\overline{\mathsf{V}}$);
4. $\exists x\, \mathsf{Nat}_n(x)$, for all $n \in \mathbb{N}$ (all individual natural numbers exist).

Here, the formulas $\mathsf{Nat}_n(x)$ expressing the fact that x is the ordinal n are defined in the usual manner: $\mathsf{Nat}_0(x)$ is $\forall y\, y \notin x$ and $\mathsf{Nat}_{n+1}(x)$ is $\forall y\,(\mathsf{Nat}_n(y) \to \forall z(z \in x \leftrightarrow z = y \vee z \in y))$. The intended interpretation of the function $\overline{\mathsf{V}}$ is $\overline{\mathsf{V}}\colon x \longmapsto \mathsf{V}_\alpha$, where α is least ordinal such that $x \subseteq \mathsf{V}_\alpha$.

[1] P. Pudlák, *Provability algebras and proof-theoretic ordinals*. ***The Journal of Symbolic Logic***, vol. 50 (1985), no. 2, pp. 423–441.

[2] D. E. Willard, *A generalization of the second incompleteness theorem and some exceptions to it*. ***Annals of Pure and Applied Logic***, vol. 141 (2006), no. 3, pp. 472–496.

▸ ALBERT VISSER, *Löb's logic and the Lewis arrow*.
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My talk reports on research in collaboration with Tadeusz Litak.

In the constructive context, the Lewis arrow does not reduce to the modal box. Moreover, a slight generalization of the Lewis arrow, has contraposed interpretability as a special case.

I will discuss versions of Löb's logic with the Lewis arrow. I will address:

- the definition of various systems,
- Kripke semantics,
- explicit fixed points,
- uniform interpolation (which is at present only known for two special systems),
- arithmetical interpretations.

At the end of the talk, I will briefly present some questions for further research.

Abstracts of invited talks in the Special Session on Set Theory

▸ YAIR HAYUT, *Stationary Reflection at the successor of a singular cardinal*.
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In the article [2], the consistency of stationary reflection at all stationary subset of $\aleph_{\omega+1}$ which concentrate on ordinals of uncountable cofinality, was obtained from the existence of a cardinal κ which is κ^+-supercompact. Using a similar method, Zeman showed in [5] that $\neg\square_{\aleph_\omega}$ is consistent relative to the weaker assumption—a measurable subcompact cardinal. In both cases, Prikry forcing is used in order to singularize a measurable cardinal that will become the new $\aleph_\omega$. When trying to improve those results in order to obtain full stationary reflection at $\aleph_{\omega+1}$ one needs to deal with the nonreflecting stationary sets which are introduced by the Prikry forcing.

In this talk, I will describe the main ideas behind the method which is used in a joint work with Spencer Unger, [4]. In this work we obtain full stationary reflection at $\aleph_{\omega+1}$, starting from a large cardinal axiom weaker than the one from [2]. This method uses the ideas of [1] and [3], and enables us to analyse the properties of a Prikry type generic extensions by using internal analysis of some iterated ultrapowers, as well as construct a specialized Prikry type forcing notion with a controlled behaviour for our problem.

[1] L. Bukovský, *Iterated ultrapower and Prikry's forcing*. ***Commentationes Mathematicae Universitatis Carolinae***, vol. 18 (1977), no. 1, pp.77–85.

[2] J. Cummings, M. Foreman, and M. Magidor, *Squares, scales and stationary reflection*. ***Journal of Mathematical Logic***, vol. 1 (2001), no. 1, pp.35–98.

[3] P. Dehornoy, *Iterated ultrapowers and Prikry forcing*. ***Annals of Mathematical Logic***, vol. 15 (1978), no. 2, pp.109–160.

[4] Y. Hayut and S. Unger, *Stationary Reflection*, preprint, 2018, arXiv:1804.11329.

[5] M. Zeman, *Two upper bounds on consistency strength of $\neg\square_{\aleph_\omega}$ and stationary set reflection at two successive $\aleph_n$*. ***Notre Dame Journal of Formal Logic***, vol. 58 (2017), no. 3, pp. 409–432.

► HEIKE MILDENBERGER AND SAHARON SHELAH, *Generalised Miller forcing may collapse cardinals*.
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We show that it is independent whether club-κ-Miller forcing preserves κ^{++}. With club guessing and other prediction principles we show that under $\kappa^{<\kappa} > \kappa$, club-$\kappa$-Miller forcing collapses $\kappa^{<\kappa}$ to κ. We investigate variants of κ-Miller forcing and draw connections to the forcing $([\kappa]^\kappa, \subseteq)$.

► DANIEL T. SOUKUP, *Through the lense of uniformization*.
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The main goal of this talk is to review recent applications of the uniformization property of ladder systems on ω_1. This notion played a critical role in S. Shelah's solution of the Whitehead problem; in the understanding of forcing axioms which can be consistent with CH [2]; and in J. Moore's work on minimal uncountable linear orders [1]. We shall focus on more recent results concerning edge colourings of graphs with uncountable chromatic number (joint work with M. Dzamonja, T. Inamdar, and J. Steprans) and questions about minimal uncountable linear orders [5, 6]. The latter topic leads to the analysis of uniformizations on Aronszajn trees [3, 4] which we shall touch on briefly.

[1] J. Moore, *ω_1 and $-\omega_1$ may be the only minimal uncountable linear orders*. ***The Michigan Mathematical Journal***, vol. 55 (2007), no. 2, pp. 437–457.

[2] S. Shelah, ***Proper Forcing***, Springer, Berlin, Heidelberg, 1982.

[3] D. T. Soukup, *Ladder system uniformization on trees I: Colouring ladders*. ***Fundamenta Mathematicae***, submitted, https://arxiv.org/abs/1806.03867.

[4] ———, *Ladder system uniformization on trees II: Growing trees*. ***Fundamenta Mathematicae***, submitted, https://arxiv.org/abs/1806.03867.

[5] ———, *A model with Suslin trees but no minimal uncountable linear orders other than ω_1 and $-\omega_1$*. ***Israel Journal of Mathematics***, to appear, https://arxiv.org/abs/1803.03583.

[6] ———, *Uncountable strongly surjective linear orders*. ***Order***, vol. 36 (2019), no. 1, pp. 43–64.

► ANDY ZUCKER, *Bernoulli disjointness*.
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We consider the concept of disjointness for topological dynamical systems, introduced by Furstenberg. We show that for every discrete group, every minimal flow is disjoint from the Bernoulli shift. We apply this to give a negative answer to the Ellis problem for all such groups. For countable groups, we show in addition that there exists a continuum-sized family of mutually disjoint free minimal systems. Using this, we can identify the underlying space of the universal minimal flow of every countable group, generalizing results of Balcar–Błaszczyk and Turek. In the course of the proof, we also show that every countable ICC group admits a free minimal proximal flow, answering a question of Frisch, Tamuz, and Vahidi Ferdowsi.

Acknowledgment. This is joint work with Eli Glasner, Todor Tsankov, and Benjamin Weiss.

Abstracts of contributed talks

▸ RYOTA AKIYOSHI AND ANDREW ARANA, *On Gaisi Takeuti's philosophy of mathematics*.
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Gaisi Takeuti (1926–2017) is one of the most distinguished logician in proof theory after Hilbert and Gentzen. He furthered the realization of Hilbert's program by formulating Gentzen's sequent calculus for higher-oder logics, conjecturing the cut-elimination theorem holds for it (Takeuti's conjecture), and obtaining several stunning results in the 1950–60's towards the solution of his conjecture. Though he has been chiefly known as a great mathematician, he wrote many articles in English and Japanese [2, 3, 4] where he expressed his philosophical thoughts.

In this talk, we aim to describe a general outline of our project to investigate Takeuti's philosophy of mathematics. In particular, we point out that there is a crucial difference between Takeuti's program and Hilbert's program, which is based on the fact that Takeuti's philosophical thinking goes back to Nishida's philosophy in Japan.

[1] R. Akiyoshi and A. Arana, *Takeuti's proof theory in the context of the Kyoto School*, ***Jahrbuch für Philosophie das Tetsugaku-Ronso***, Kyoto University, to appear.

[2] ———, *Keishikisyugi No Tachiba Kara II* (*From the point of view of formalism II*). ***Kagakukisoron Kenkyu* (*Annals of the Japan Association for Philosophy of Science*)**, vol. 2 (1956), no. 3, pp. 295–299.

[3] ———, *Suugaku Ni Tsuite* (*About mathematics*). ***Kagakukisoron Kenkyu* (*Annals of the Japan Association for Philosophy of Science*)**, (1972) no. 4, pp. 170–174.

[4] ———, ***Proof-Theory***, second ed., North-Holland, Amsterdam, 1987.

▸ SENIK ALVRTSYAN, SERGEY DAVIDOV, AND DAVIT SHAHNAZARYAN, *Invertible binary algebras principally isotopic to a group*.
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A binary groupoid $Q(A)$ is a nonempty set Q together with a binary operation A. Binary groupoid $Q(A)$ is called quasigroup if for all ordered pairs $(a, b) \in Q^2$ exists unique solutions $x, y \in Q$ of the equations $A(a, x) = b$ and $A(y, a) = b$. The solutions of these equations will be denoted by $x = A^{-1}(a, b)$ and $y = {}^{-1}A(b, a)$, respectively. A binary algebra $(Q; \Sigma)$ is called invertible algebra or system of quasigroups if each operation in Σ is a quasigroup operation.

We obtained characterizations of invertible algebras isotopic to a group or an abelian group by the second-order formula.

Definition 1. We say that a binary algebra $(Q; \Sigma)$ is isotopic to the groupoid $Q(\cdot)$, if each operation in Σ is isotopic to the groupoid $Q(\cdot)$, that is, for every operation $A \in \Sigma$ there exists permutations $\alpha_A, \beta_A, \gamma_A$ of Q, that:

$$\gamma_A A(x, y) = \alpha_A x \cdot \beta_A y, s$$

for every $x, y \in Q$. Isopoty is called principal if $\gamma_A = epsilon(\epsilon$ - unit permutation) for every $A \in \Sigma$.

THEOREM 2. *The invertible algebra $(Q;\Sigma)$ is a principally isotopic to the abelian group, if and only if the following second-order formula*

$$A({}^{-1}A(B(x,B^{-1}(y,z)),u),v) = B(x,B^{-1}(y,A({}^{-1}A(z,u),v))),$$

is valid in the algebra $(Q;\Sigma\cup\Sigma^{-1}\cup{}^{-1}\Sigma)$ for all $A,B\in\Sigma$.

COROLLARY 3. [1] *The class of quasigroups isotopic to groups is characterized by the following identity*:

$$x(y\backslash((z/u)v)) = ((x(y\backslash z))/u)v.$$

THEOREM 4. *The invertible algebra $(Q;\Sigma)$ is a principally isotopic to a group if and only if the following second-order formula*:

$$A({}^{-1}A(B(x,z),y),A^{-1}(u,B(w,y))) =$$

$$= A({}^{-1}A(B(w,z),y),A^{-1}(u,B(x,y))),$$

is valid in the algebra $(Q;\Sigma\cup\Sigma^{-1}\cup{}^{-1}\Sigma)$ for all $A,B\in\Sigma$.

COROLLARY 5. *The class of quasigroups isotopic to abelian groups is characterized by the following identity*:

$$((xz)/y)(u\backslash(wy)) = ((wz)/y)(u\backslash(xy)).$$

[1] V. D. BELOUSOV, *Globaly associative systems of quasigroups*. ***Matematicheskii Sbornik***, vol. 55 (1961), nos. 2, 97, pp. 221–236.

► MAHFUZ RAHMAN ANSARI AND A. V. RAVISHANKAR SARMA, *Constraints on selection function*: *A critique of Lewis-Stalnaker's semantics for counterfactuals*.
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Counterfactual conditionals are the special kind of conditional sentences $P\Box\rightarrow Q$, in which the antecedent is always false. Counterfactual conditionals are statements, asserting that something happens under certain conditions, which are presupposed not to be satisfied in reality. The semantics of counterfactuals has been a challenging task for philosophers, since antiquity. The most celebrated and poplar approach in this direction is the Stalnaker (1968)-Lewis (1973) possible-worlds semantics. According to Lewis-Stalnaker' semantics, a counterfactual $P\Box\rightarrow Q$ holds when in the nearest possible world with respect to the antecedent, the consequent is also true. This approach is based on the comparative similarity of possible worlds. Despite its mathematical elegance, this approach is not free from problems. There is a gap between intuitive notion of similarity of possible worlds and the criteria provided by Lewis. In this article, we restrict ourselves to the counterfactual conditionals in which the antecedents are treated as action deliberations. We emphasize on additional constraints that are to be imposed on the selection function that picks the nearest possible world. The present study aims to explore the constraints on selection function and tries to reduce the gap between intuitive understanding of counterfactuals and formal analysis of counterfactuals, based on similarity of possible worlds.

[1] M. L. GINSBERG, *Counterfactuals*. ***Artificial Intelligence***, vol. 30 (1986), no. 1, pp. 35–79.

[2] D. LEWIS, ***Counterfactuals***, Harvard University Press, Cambridge, MA, reissued, Blackwell, London, 2001.

[3] ———, ***On the Plurality of Worlds***, Oxford, Basil Blackwell, 1986.

[4] D. NUTE, *Counterfactuals and the similarity of words*. ***The Journal of Philosophy***, vol. 72 (1975), no. 21, pp. 773–778.

[5] R. STALNAKER, *A theory of conditionals in studies in logical theory*, ***American Philosophical Quarterly Monograph Series 2***, Oxford, Blackwell, 1986, pp. 98–112.

▸ JAMES APPLEBY, *Resolving two paradoxes about knowledge states in the foundations of intuitionistic analysis*.
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A choice sequence is a continually growing sequence whose growth may, or may not, be restricted in some way. They were utilised by Brouwer to resolve a crucial issue with his intuitionistic re-foundation of mathematics; specifically, they allowed him to bridge gap between the rationals and the reals.

Choice sequences received no true formalisation in Brouwer's works, however, from [2] onwards, he considered them as a pair of growing objects; a list of elements generated so far, and a list of intensional first order restrictions.

A knowledge state is a formalised way of representing finite information about choice sequences. This allows us to formally represent intensional information about choice sequences and achieve a notion of choice sequence close to that proposed by Brouwer. The theory *FIM-KS* put forward in [3] demonstrates that knowledge states can be used to successfully found intuitionistic analysis. They have also been used in [1] to show that the theory of the creating subject is not needed.

This talk demonstrates that the theory of knowledge states put forward in [3] allows two paradoxes to be derived, and it then outlines their resolution.

[1] J. F. Appleby, ***Choice Sequences and Knowledge States*: *Extending the Notion of Finite Information to Produce a Clearer Foundation for Intuitionistic Analysis***, Doctoral Thesis, Keele University, 2017.

[2] L. E. J. Brouwer, *Zur begrundung der intuitionistischen Mathematik I*. ***Mathematische Annalen***, vol. 93 (1925), pp. 244–257.

[3] P. Fletcher, *Brouwer's weak counterexamples and the creative subject*: *A critical survey*. ***Journal of Philosophical Logic***, upcoming.

▸ TOSHIYASU ARAI, *Some results in proof theory*.
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Let me report on some recent results in proof theory such as the proof-theoretic strengths of the well-ordering principles and of reflecting ordinals.

▸ PHILIPPE BALBIANI AND TINKO TINCHEV, *Computability of contact logics with measure*.
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Contact logics [1] are propositional logics interpreted over Boolean contact algebras [3]. They stem from the point-free approaches of geometry put forward by Whitehead. Their language $\mathcal{L}(\leq, C)$ includes Boolean terms representing regions. Let $\mathbf{X}$ be a set of variables. The set of Boolean terms (s, t, etc) over $\mathbf{X}$ being denoted $\mathbf{T}(\mathbf{X})$, the set $\mathbf{A}(\mathbf{X})$ of atomic formulas over $\mathbf{X}$ consists of all expressions of the form $s \leq t$ ("s is part-of t") and $C(s,t)$ ("s is in contact with t"). The set of all formulas (φ, ψ, etc) over $\mathbf{X}$ is the least set $\mathbf{F}(\mathbf{X})$ containing $\mathbf{A}(\mathbf{X})$ and such that for all $\varphi, \psi \in \mathbf{F}(\mathbf{X})$: $\perp \in \mathbf{F}(\mathbf{X})$, $\neg\varphi \in \mathbf{F}(\mathbf{X})$ and $(\varphi \vee \psi) \in \mathbf{F}(\mathbf{X})$. Of interest are, of course, the sets of all valid formulas determined by the various classes of Boolean contact algebras one may consider. See [1, 5] for detailed investigations.

The combination of topological and size information is a fundamental issue for multifarious applications of spatial reasoning [4]. It can be realized by considering Boolean contact algebras with measure, that is, algebraic structures (A, C, μ) where (A, C) is a Boolean contact algebra and μ is a positive finite measure on A. Contact logics with measure are extensions of contact logics. Their language $\mathcal{L}(\leq, C, \leq_m)$ contains all additional atomic formulas of the form $s \leq_m t$ ("the size of s is less or equal than the size of t"). Of interest are, again, the sets of all valid formulas determined by the various classes of Boolean contact algebras with measure one may consider.

Using complexity results about linear programming [2], we show that the set of all valid formulas determined by the class of all Boolean contact algebras with measure is in *coNP*. Our proof relies on the equivalence between the satisfiability of a given formula φ and the consistency of an associated system $\mathcal{S}_\varphi$ of linear inequalities. It uses the following facts: the computation of $\mathcal{S}_\varphi$ from φ is possible in nondeterministic polynomial time; if a system of k linear inequalities with integer coefficients of length at most n has a nonnegative solution then it has a nonnegative solution with at most k positive entries of length in $\mathcal{O}(k.(n + \log k))$.

[1] P. Balbiani, T. Tinchev, and D. Vakarelov, *Modal logics for region-based theories of space*. ***Fundamenta Informaticæ***, vol. 81 (2007), pp. 29–82.

[2] V. Chvátal, ***Linear Programming***, Freeman, 1983.

[3] G. Dimov and D. Vakarelov, *Contact algebras and region-based theory of space: A proximity approach–I*. ***Fundamenta Informaticæ***, vol. 74 (2006), pp. 209–249.

[4] A. Gerevini and J. Renz, *Combining topological and size information for spatial reasoning*. ***Artificial Intelligence***, vol. 137 (2002), pp. 1–42.

[5] R. Kontchakov, I. Pratt-Hartmann, and M. Zakharyaschev, *Spatial reasoning with RCC8 and connectedness constraints in Euclidean spaces*. ***Artificial Intelligence***, vol. 217 (2014), pp. 43–75.

▸ JOHN BALDWIN, *On strongly minimal Steiner systems: Zilber's conjecture, universal algebra, and combinatorics*.

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With Gianluca Paolini [1], we constructed, using a variant on the Hrushovski dimension function, for every $k \geq 3$, 2^μ families of strongly minimal Steiner k-systems. We study the mathematical properties of these counterexamples to Zilber's trichotomy conjecture rather than thinking of them as merely exotic examples. In particular, the long study of finite Steiner systems is reflected in results that depend on the block size k. A quasigroup is a structure with a binary operation such that for each equation $xy = z$ the values of two of the variables determines a unique value for the third. The new Steiner 3-systems are bi-interpretable with strongly minimal Steiner quasigroups. For $k > 3$, we expect to show the pure k-Steiner systems have 'essentially unary definable closure' and do not interpret a quasigroup. But, we show that for q a prime power the Steiner q-systems can be interpreted into specific sorts of quasigroups, block algebras. This show a dichotomy within the class of strongly minimal sets with flat geometries.

We extend the notion of an (a, b)-cycle graph arising in the study of finite and infinite Stein triple systems [2] by introducing what we call the (a, b)-path graph of a block algebra. We exhibit theories of strongly minimal block algebras where all (a, b)-paths are infinite and others in which all are finite only in the prime model. We show how to obtain combinatorial properties (e.g., 2-transitivity) by either varying the basic collection of finite partial Steiner systems or modifying the μ function which ensures strong minimality.

[1] J. T. Baldwin and G. Paolini, *Strongly minimal Steiner systems I*, submitted, 2018.

[2] P. J. Cameron and B. S. Webb, *Perfect countably infinite Steiner triple systems*. ***The Australasian Journal of Combinatorics***, vol. 54 (2012), pp. 273–278.

▸ NIKOLAY BAZHENOV, HRISTO GANCHEV, AND STEFAN VATEV, *Computable embeddings for pairs of linear orderings*.

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Friedman and Stanley [3] introduced the notion of *Borel embedding* to compare complexity of the classification problems for classes of countable structures. Calvert, Cummins, Knight, and Miller [1] (see also [2] and [4]) developed two notions, *computable embeddings* and *Turing computable embeddings*, as effective counterparts of Borel embeddings.

We follow the approach of [1] and study computable embeddings for pairs of structures, that is, for classes $\mathcal{K}$ containing precisely two nonisomorphic structures. Our motivation for investigating pairs of structures is two-fold. These pairs play an important role in computable structure theory and also they constitute the simplest case, which is significantly different from the case of one-element classes. It is not hard to show that for any computable structures $\mathcal{A}$ and $\mathcal{B}$, the one-element classes $\{\mathcal{A}\}$ and $\{\mathcal{B}\}$ are equivalent with respect to computable embeddings. On the other hand, computable embeddings induce a nontrivial degree structure for two-element classes consisting of computable structures.

In this talk, we will concentrate on the pair of linear orders ω and $\omega^\star$. By $\deg_{tc}(\{\omega, \omega^\star\})$ we denote the degree of the class $\{\omega, \omega^\star\}$ under Turing computable embeddings. Quite unexpectedly, it turns out that a seemingly simple problem of studying computable embeddings for classes from $\deg_{tc}(\{\omega, \omega^\star\})$ requires developing new techniques.

We give a necessary and sufficient condition for a pair of structures $\{\mathcal{A}, \mathcal{B}\}$ to belong to $\deg_{tc}(\{\omega, \omega^\star\})$. We also show that the pair $\{1 + \eta, \eta + 1\}$ is the greatest element inside $\deg_{tc}(\{\omega, \omega^\star\})$, with respect to computable embeddings. More interestingly, we prove that inside $\deg_{tc}(\{\omega, \omega^\star\})$, there is an infinite chain of degrees induced by computable embeddings.

[1] W. Calvert, D. Cummins, J. F. Knight, and S. Miller, *Comparing classes of finite structures*. ***Algebra Logic***, vol. 43 (2004), no. 6, pp. 374–392.

[2] J. Chisholm, J. F. Knight, and S. Miller, *Computable embeddings and strongly minimal theories*. ***The Journal of Symbolic Logic***, vol. 72 (2007), no. 3, pp. 1031–1040.

[3] H. Friedman and L. Stanley, *A Borel reducibility theory for classes of countable structures*. ***The Journal of Symbolic Logic***, vol. 54 (1989), no. 3, pp. 894–914.

[4] J. F. Knight, S. Miller, and M. V. Boom, *Turing computable embeddings*. ***The Journal of Symbolic Logic***, vol. 72 (2007), no. 3, pp. 901–918.

► NIKOLAY BAZHENOV, MANAT MUSTAFA, AND MARS YAMALEEV, *Computable reducibility, and isomorphisms of distributive lattices*.

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A standard tool for classifying computability-theoretic complexity of equivalence relations is provided by computable reducibility. Let E and F be equivalence relations on ω. The relation E is *computably reducible* to F, denoted by $E \leq_c F$, if there is a total computable function $f(x)$ such that for all $x, y \in \omega$,

$$(x\,E\,y) \;\Leftrightarrow\; (f(x)\,F\,f(y)).$$

The systematic study of computable reducibility was initiated by Ershov [1, 2].

Let α be a computable nonzero ordinal. An equivalence relation R is Σ^0_α *complete* (for computable reducibility) if $R \in \Sigma^0_\alpha$ and for any Σ^0_α equivalence relation E, we have $E \leq_c R$. The article [4] provides many examples of Σ^0_n complete equivalence relations, which arise in a natural way in recursion theory. In [3], it was proved that for each of the following classes K, the relation of computable isomorphism for computable members of K is Σ^0_3 complete: trees, equivalence structures, and Boolean algebras.

We prove that for any computable successor ordinal α, the relation of Δ^0_α isomorphism for computable distributive lattices is $\Sigma^0_{\alpha+2}$ complete. We obtain similar results for Heyting algebras, undirected graphs, and uniformly discrete metric spaces.

[1] Y. L. Ershov, *Positive equivalences*. ***Algebra and Logic***, vol. 10 (1971), no. 6, pp. 378–394.

[2] ———, ***Theory of Numberings***, Nauka, Moscow, 1977, (in Russian).

[3] E. Fokina, S.-D. Friedman, and A. Nies, *Equivalence relations that are Σ^0_3 complete for computable reducibility*, ***Logic, Language, Information and Computation*** (L. Ong and R. de Queiroz, editors), Lecture Notes in Computer Science, vol. 7456, Springer, Berlin, 2012, pp. 26–33.

[4] E. Ianovski, R. Miller, K. M. Ng, and A. Nies, *Complexity of equivalence relations and preorders from computability theory*. ***The Journal of Symbolic Logic***, vol. 79 (2014), no. 3, pp. 859–881.

► NIKOLAY BAZHENOV, DINO ROSSEGGER, LUCA SAN MAURO, AND MAXIM ZUBKOV, *On bi-embeddable categoricity of linear orders*.
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Given a linear order $\mathcal{L}$ and a linear order $\mathcal{M}$ bi-embeddable with $\mathcal{L}$, we say that $\mathcal{M}$ is a bi-embeddable copy of $\mathcal{L}$. We study the complexity of embeddings using the following definition analogous to computable categoricity.

Definition 1. A countable linear order $\mathcal{L}$ is (relatively) Δ^0_n-bi-embeddably categorical if for any bi-embeddable computable (for any bi-embeddable) copy $\mathcal{M}$, $\mathcal{M}$ and $\mathcal{L}$ are bi-embeddable by Δ^0_n-embeddings ($\Delta^{\mathcal{L}\oplus\mathcal{M}}_n$-embeddings, correspondingly).

Recall, that a linear order is scattered if it has no a suborder of type η. It is easy to see, that the question about the level of bi-embeddable categoricity is nontrivial only for scattered linear orders. We obtain characterization of linear orders with finite levels of bi-embeddable categoricity.

Theorem 2. *A scattered computable linear order of rank n is relatively Δ^0_{2n}-bi-embeddably categorical, and is not Δ^0_{2n-1}-bi-embeddably categorical.*

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► FRODE ALFSON BJØRDAL, *Capture, Replacement, Specification*.
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Let **W** be Zermelo set theory **Z** minus specification and choice. For $\alpha(v, x, y)$ any first order condition in the language of set theory on the indicated free variables, legislate:

Axiom of Capture:

$\forall v \exists w \forall x (x \in w \leftrightarrow \exists y (y \in v \wedge \alpha(v, x, y) \wedge (\forall z)(\alpha(v, z, y) \rightarrow x = z)))$

Let **ZF** be Zermelo-Fraenkel set theory: We show $\mathbf{ZF} = \mathbf{W} + \textit{Axiom of Capture}$. Capture avoids the cumbersome restriction to *functional* condition, and is justified by the idea that we should accept as many instances of naive comprehension as possible. Versions of capture are of use in the context of the author's alternative set theory £ as in [1] because they allow for more flexibility in expressing useful closure principles.

[1] F. A. Bjørdal, *Elements of librationism*, http://arxiv.org/abs/1407.3877.

► PIOTR BŁASZCZYK, *Axioms for Euclid's Elements book V, their consequences and some independence results*.
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Euclid's *Elements*, book V develops the theory of proportions as applied to magnitudes; it is the key theory for understanding Greek and early modern mathematics. By formalizing its definitions and the tacit assumptions behind its proofs, we reconstruct book V with its 25 Propositions as an axiomatic theory.

The general term $\mu\varepsilon\gamma\varepsilon\theta o\varsigma$ covers line segments, triangles, convex polygones, circles, solids, angles, and arcs of circles. We formalize Euclid's magnitudes of the same kind (line segments being of one kind, triangles being of another, etc.) as an additive semigroup with a total order, $(M, +, <)$, characterized by the following five axioms:

E1. $(\forall x, y)(\exists n \in \mathbb{N})[nx > y]$,
E2. $(\forall x, y)(\exists z)[x < y \Rightarrow x + z = y]$,
E3. $(\forall x, y, z)[x < y \Rightarrow x + z < y + z]$,
E4. $(\forall x)(\forall n \in \mathbb{N})(\exists y)[ny = x]$,
E5. $(\forall x, y, z)(\exists v)[x : y :: z : v]$.

We show that E4 follows from E1–E3, E5; we prove the independence of the Axioms E1, E2, E3. We discuss the use of E1 in the Proposition V.8; we show that E1 does not follow from the Dedekind completeness axiom (although it does follow from the completeness axiom in an orderd group). We interpret Greek proportion in an Archimedean ordered field, and offer an algebraic interpretation of the axiom *The whole is greater than the part*.

We present schemes of Euclid's propositions; they consist of algebraic formulae representing sequences of (grammatical) sentences, signs representing phrases that occur in the Greek text and references to the axioms, definitions, and other propositions. We discuss under what assumptions these schemes could be turned into modern proofs. Finally, we present algebraic paraphrases of all 25 Propositions of book V as derived from the Axioms E1–E5.

[1] F. BECKMANN, *Neue Gesichte zum 5. Buch Euklidis*. ***Archive for History of Exact Sciences***, vol. 4 (1967), pp. 1–144.

[2] P. BŁASZCZYK and K. MRÓWKA, ***Euklides, Elementy, Ksiegi V-VI. Tłumaczenie i Komentarz***, Copernicus Center Press, 2013.

[3] J. L. HEIBERG, ***Euclidis Elementa***, Teubneri, 1883–1888.

[4] I. MUELLER, ***Philosophy of Mathematics and Deductive Structure in Euclid's Elements***, MIT Press, 2006.

► PIOTR BŁASZCZYK AND MARLENA FILA, *Limits of diagrammatic reasoning*.
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We challenge theses of [2] and [4] concerning the Intermediate Value Theorem (IVT); we argue that a diagrammatic reasoning is reliable provided one finds a formula representing the diagram.

IVT states: If $(F, +, \cdot, 0, 1, <)$ is an ordered field, $f\colon [0, 1] \mapsto F$ is a continuous map such that $f(0)f(1) < 0$, then $f(x) = 0$, for some $x \in (0, 1)$. An accompanying diagram, diag(IVT), depicts a graph of f intersecting a line $(F, <)$, as the function values differ in sign.

(a) In [2], Brown argues that diag(IVT) guarantees the existence of an intersection point. (b) In [4], Giaquinto argues that diag(IVT) do not guarantee the existence thesis, since continuous functions include nonsmooth functions that find no graphic representations.

(ad a) We show that IVT is equivalent to Dedekind Cuts principle (DC): If (A, B) is a Dedekind cut in $(F, <)$, then

$$(\exists! c \in F)(\forall x \in A)(\forall y \in B)[x \leq c \leq y].$$

We also provide a graphic representation for DC.

This equivalence justifies the claim that IVT is as obvious as DC. There is, however, no relation between diag(IVT) and diag(DC), all the more between diag(IVT) and the formula DC. Thus, Brown's claim has to be based on the analytic truth $IVT \Leftrightarrow DC$.

(ad b) Diagrams representing lines $(F, <)$ do not depict whether the field $(F, +, \cdot, 0, 1, <)$ is Euclidean (closed under the square root operation), or $(\mathbb{R}, +, \cdot, 0, 1, <)$, or a real-closed field; graphs of f do not distinguish between polynomial and smooth functions. IVT for polynomials, IVT_p, is valid in real-closed fields (these fields could be *bigger* or *smaller* than real numbers); in fact, IVT_p is the axiom for real-closed fields (next to the Euclidean condition).

Bolzano is believed to give the first proof of IVT. In fact, he sought to prove IVT_p, whilst IVT was just the lemma. Mislead by a diagram, Bolzano proved the theorem not as general as it could be: he proved only that IVT_p is valid in the domain of real numbers.

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[2] B. Bolzano, ***Rein Analytischer Beweis***, Gotlieb Hasse, 1817.

[3] J. R. Brown, *Proofs and pictures*. ***The British Journal for the Philosophy of Science***, vol. 48 (1997), pp. 161–180.

[4] M. Giaquinto, *Crossing curves*: *A limit to the use of diagrams in proofs*. ***Philosophia Mathematica***, vol. 19 (2011), pp. 181–207.

► MARIJA BORIČIĆ, *Suppes–style natural deduction system for classical logic*.
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An elegant way to work with probabilized sentences was proposed by P. Suppes (see [3] and [4]). According to his approach we develop a natural deduction system $NKprob(\varepsilon)$ inspired by Gentzen's natural deduction system *NK* for classical propositional logic. We use a similar approach as in defining general probability natural deduction system *NKprob* (see [1]). Our system will be suitable for manipulating sentences of the form A^n, where A is any propositional formula and n a natural number, with the intended meaning 'the probability of truthfulness of A is greater than or equal to $1 - n\varepsilon$', for some small $\varepsilon > 0$.

For instance, the rules dealing with conjunction looks as follows:

$$\frac{A^m \quad B^n}{(A \wedge B)^{m+n}}(I\wedge) \qquad \frac{A^m \quad (A \wedge B)^n}{B^n}(E\wedge)$$

and modus ponens:

$$\frac{A^m \quad (A \to B)^n}{B^{m+n}}$$

The system $NKprob(\varepsilon)$ will be a natural counterpart of our sequent calculus $LKprob(\varepsilon)$ (see [2]).

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[3] P. Suppes, *Probabilistic inference and the concept of total evidence*, ***Aspects of Inductive Logic*** (J. Hintikka and P. Suppes, editors), North-Holland, Amsterdam, 1966, pp. 49–65.

[4] C. G. Wagner, *Modus tollens probabilized*. ***British Journal for the Philosophy of Science***, vol. 54 (2004), no. 4, pp. 747–753.

► DAVID BRADLEY-WILLIAMS, *Canonical invariants for t-stratifications*.
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A classical tool in singularity theory is the notion of a stratification of algebraic subsets of $\mathbb{R}^n$ or $\mathbb{C}^n$. In [1], Immanuel Halupczok has developed the notion of *t-stratification* in the context of sets definable in a valued field. We will present joint work with I. Halupczok, in which we investigate invariants of such stratifications that we associate canonically to definable sets, with particular interest in valued fields such as such as $\mathbb{R}((t))$ and $\mathbb{C}((t))$.

Acknowledgment. This is joint work with Immanuel Halupczok (HHU Düsseldorf).

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▸ ANAHIT CHUBARYAN AND ARTUR KHAMISYAN, *On the proof complexity in two universal proof system for all versions of many-valued logics*.

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Two types of universal propositional proof systems were described in [2] such that propositional proof system for every version of MVL can be presented in both of described forms. The first of introduced systems (US) is a Gentzen-like system, the second one (UE) is based on the generalization of the notion of determinative disjunctive normal form, defined by first coauthor for two-valued logic [1]. The last type proof systems are weak ones with a simple strategist of proof search and we have investigated the quantitative properties, related to proof complexity characteristics in them. In particular, for some class of many-valued tautologies simultaneously optimal bounds (asymptotically the same upper and lower bounds) for each of main proof complexity characteristics (size, steps, space, and width) were obtained in the second-type systems, considered for some versions of many-valued logic. Now we investigate the relations between the main proof complexty measures in both universal systems. We prove that the system UE *p-simulates* the system US, but the system US does not *p-simulate* the system UE and, therefore, the systems UE and US do not be *p-equivalent*, but nevertheless some classes of k-tautologies have the same proof complexities bounds in both systems, hence we obtain symilar results in Gentzen-like system for the same and for other classes of many-valued tautologies as well.

Acknowledgment. This work was supported by the RA MES State Committee of Science, in the frames of the research project Nr. 18T-1B034.

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▸ ANAHIT CHUBARYAN, GARIK PETROSYAN, AND SERGEY SAYADYAN, *Monotonous and strong monotonous properties of some propositional proof systems for classical and non classical logics*.

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For some propositional proof system of classical and nonclassical logics we investigate the relations between the lines (*t-complexities*) and sizes (*l-complexities*) of proofs for minimal tautologies, which are not a substitution of a shorter tautology of this logic, and results of a substitutions in them. For every minimal tautology φ of fixed logic by $S(\varphi)$ is denoted the set of all tautologies, which are results a substitution in φ.

Definition 1. The proof system Φ is called *t-monotonous* (*l-monotonous*), if for every minimal tautology φ of this system and for every formula ψ from $S(\varphi)$ $t^{\Phi}(\varphi) \leq t^{\Phi}(\psi)$ ($l^{\Phi}(\varphi) \leq t^{\Phi}(\psi)$).

DEFINITION 2. The proof system Φ is called *t-strong monotonous* (*l-strong monotonous*), if for every nonminimal tautology ψ of this system there is such minimal tautology φ of this system such that ψ belong to $S(\varphi)$ and $t^{\Phi}(\psi) \leq t^{\Phi}(\varphi)$ ($l^{\Phi}(\psi) \leq t^{\Phi}(\varphi)$).

Formerly it is proved in [3], that Frege systems for classical and nonclassical logics are neither *t-monotonous* nor *l-monotonous*.

Now we consider the following systems: propositional resolution systems RC, RI, RJ for classical, intuitionistic and Johansson's logics accordingly, eliminations systems $E?$, EI, EJ, based on the determinative normal forms for the same logics [1], and the system GS, based on generalization of splitting method [2].

THEOREM 3. *The systems RC, RI and RJ are t-strong monotonous (l-strong monotonous), but neither of them is t-monotonous (l-monotonous).*

THEOREM 4. *Each of the systems EC, EI, EJ and GS is neither t-monotonous (l-monotonous) nor t-strong monotonous (l-strong monotonous).*

Acknowledgment. This work was supported by the RA MES State Committee of Science, in the frames of the research project Nr. 18T-1B034.

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[3] A. CHUBARYAN and G. PETROSYAN, *Frege systems are no monotonous*. ***Evolutio***, vol. 3 (2016), pp. 12–14.

► LUDOVICA CONTI, *One or more* Logicisms.
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The aim of this talk consists in comparing different ways to pursue a logicist project. More in particular, I would compare a proof-theoretic version of logicism, like Tennant's *costructivist logicism* (CL [3]), with two axiomatic versions, namely Heck's *finite Frege Arithmetic* (FFA [1]) and a *free zig-zag logicism* (FZL), obtained by the adoption of a negative free logic and a restricted version of Basic Law V[1].

Both these three systems allows us to derive any instance of the comprehension axiom schema but the different restrictions of the logic (in CL and in FZL) and of the abstraction principles (HP in FFA and BLV in FZL) determine the different strength of the theories.

My two aims consist in, first, discussing the conjecture (proposed by Tennant in [3]) that CL is the intuitionistic (relevant) fragment of Heck's FFA and, secondly, clarifying the existential role of abstraction principles in systems which adopt free logic. Comparing the derivational power of CL and FZL, we can observe that the first one allows us to derive the existential claim $\exists x(x = \sharp F)$ only where F is a concept with a finite extension, while the second one allows us to derive also the existential instance of such theorem where F means *natural number*—namely a concept with an infinite extension.

[1]T-BLV: $\forall F \forall G(\epsilon(F) = \epsilon(G) \leftrightarrow \bigwedge x(Fx \leftrightarrow Gx) \wedge (\phi(F) \wedge \phi(G))$—where ϕ means "positive"—it contains second-order variables only in the scope of an even number of negation symbols).

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[3] N. TENNANT, *Existence and identity in free logic: A problem for inferentialism*? ***Mind, New Series***, vol. 116 (2007), no. 464, pp. 1055–1078.

► VALERIA DE PAIVA, LUIZ CARLOS PEREIRA, AND ELAINE PIMENTEL, *New ecumenical systems*.
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Much has been said about the connections between intuitionistic logic and classical logic. Recently, Prawitz (see [4]) proposed a natural deduction *ecumenical* system that puts together classical and intuitionistic logic in a single system, a codification where classical logic and intuitionistic logic can coexist "in peace." The main idea behind this codification is that classical and intuitionist logic share the constants for conjunction, negation, the absurd, and the universal quantifier, but each has its own disjunction, implication and existential quantifier. Similar ideas are present in Dowek [2] and Krauss [3]. The aims of the present article are: (i) to present an ecumenical sequent calculus for classical and intuitionistic logic and to state some proof theoretical properties of the system, and (ii) to propose a new ecumenical system, based on the multiple conclusion intuitionistic sequent calculus FIL [1], that combines classical logic and the logic of constant domains.

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▸ RUMEN DIMITROV, VALENTINA HARIZANOV, ANDREY MOROZOV, PAUL SHAFER, ALEXANDRA SOSKOVA, AND STEFAN VATEV, *Cohesive powers of ω*.
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A *cohesive power* of a computable structure is an effective analog of an ultrapower of the structure in which a cohesive set plays the role of an ultrafilter. We study the cohesive powers of computable copies of the structure $(\omega, <)$, that is, the natural numbers with their usual order. By a *computable copy of* $(\omega, <)$, we mean a computable linear order $\mathcal{L} = (L, \prec)$ that is isomorphic to $(\omega, <)$, but not necessarily by a computable isomorphism. That is, the successor function of $\mathcal{L}$ may not be computable. Our main findings are the following. First, recall that ζ denotes the order type of the integers, that η denotes the order type of the rationals, and that $\omega + (\eta \times \zeta)$ (often also written $\omega + \zeta\eta$) is familiar as the order type of countable nonstandard models of Peano arithmetic.

1. If $\mathcal{L}$ is a computable copy of $(\omega, <)$ with a computable successor function, then every cohesive power of $\mathcal{L}$ has order type $\omega + (\eta \times \zeta)$.
2. There is a computable copy $\mathcal{L}$ of $(\omega, <)$ with a *non*computable successor function such that every cohesive power of $\mathcal{L}$ has order type $\omega + (\eta \times \zeta)$.
3. Most interestingly, there is a computable copy $\mathcal{L}$ of $(\omega, <)$ (with a necessarily noncomputable successor function) having a cohesive power that is *not* of order type $\omega + (\eta \times \zeta)$.

▸ DMITRY EMELYANOV, BEIBUT KULPESHOV, SERGEY SUDOPLATOV, *On compositions of structures and compositions of theories*.
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We consider both compositions of structures and compositions of theories and apply these compositions obtaining compositions of algebras of binary formulas [1].

Let $\mathcal{M}$ and $\mathcal{N}$ be structures of relational languages $\Sigma_{\mathcal{M}}$ and $\Sigma_{\mathcal{N}}$, respectively. We define the composition $\mathcal{M}[\mathcal{N}]$ of $\mathcal{M}$ and $\mathcal{N}$ satisfying $\Sigma_{\mathcal{M}[\mathcal{N}]} = \Sigma_{\mathcal{M}} \cup \Sigma_{\mathcal{N}}$, $M[N] = M \times N$ and the following conditions:

(1) if $R \in \Sigma_{\mathcal{M}} \setminus \Sigma_{\mathcal{N}}$, $\mu(R) = n$, then $((a_1, b_1), \ldots, (a_n, b_n)) \in R_{\mathcal{M}[\mathcal{N}]}$ if and only if $(a_1, \ldots, a_n) \in R_{\mathcal{M}}$;

(2) if $R \in \Sigma_{\mathcal{N}} \setminus \Sigma_{\mathcal{M}}$, $\mu(R) = n$, then $((a_1, b_1), \ldots, (a_n, b_n)) \in R_{\mathcal{M}[\mathcal{N}]}$ if and only if $a_1 = \cdots = a_n$ and $(b_1, \ldots, b_n) \in R_{\mathcal{N}}$;

(3) if $R \in \Sigma_{\mathcal{M}} \cap \Sigma_{\mathcal{N}}$, $\mu(R) = n$, then $((a_1, b_1), \ldots, (a_n, b_n)) \in R_{\mathcal{M}[\mathcal{N}]}$ if and only if $(a_1, \ldots, a_n) \in R_{\mathcal{M}}$, or $a_1 = \cdots = a_n$ and $(b_1, \ldots, b_n) \in R_{\mathcal{N}}$.

The theory $T = \mathrm{Th}(\mathcal{M}[\mathcal{N}])$ is called the *composition* $T_1[T_2]$ of the theories $T_1 = \mathrm{Th}(\mathcal{M})$ and $T_2 = \mathrm{Th}(\mathcal{N})$.

THEOREM 1. *If $\mathcal{M}$ and $\mathcal{N}$ have transitive automorphism groups then $\mathcal{M}[\mathcal{N}]$ has a transitive automorphism group, too.*

By Theorem 1, $T = \mathrm{Th}(\mathcal{M}[\mathcal{N}])$ is transitive, and the operation $\mathcal{M}[\mathcal{N}]$ can be considered as a variant of transitive arrangements of structures [2].

The composition $\mathcal{M}[\mathcal{N}]$ is called *E-definable* if $\mathcal{M}[\mathcal{N}]$ has an $\emptyset$-definable equivalence relation E whose E-classes are universes of the copies of $\mathcal{N}$ forming $\mathcal{M}[\mathcal{N}]$. By the definition, each E-definable composition $\mathcal{M}[\mathcal{N}]$ is represented as a E-combination [3] of copies of $\mathcal{N}$ with an extra-structure generated by predicates on $\mathcal{M}$ and linking elements of the copies of $\mathcal{N}$.

THEOREM 2. *If a composition $\mathcal{M}[\mathcal{N}]$ is E-definable then the theory* $\mathrm{Th}(\mathcal{M}[\mathcal{N}])$ *uniquely defines the theories* $\mathrm{Th}(\mathcal{M})$ *and* $\mathrm{Th}(\mathcal{N})$, *and vice versa.*

THEOREM 3. *If a composition $\mathcal{M}[\mathcal{N}]$ is E-definable then the algebra $\mathfrak{P}_T$ of binary isolating formulas for $T = \mathrm{Th}(\mathcal{M}[\mathcal{N}])$ is isomorphic to the composition $\mathfrak{P}_{T_1}[\mathfrak{P}_{T_2}]$ of the algebras $\mathfrak{P}_{T_1}$ and $\mathfrak{P}_{T_2}$ of binary isolating formulas for $T_1 = \mathrm{Th}(\mathcal{M})$ and $T_2 = \mathrm{Th}(\mathcal{N})$.*

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► JOSÉ ESPÍRITO SANTO AND GILDA FERREIRA, *An embedding of* **IPC** *into* $\mathbf{F_{at}}$ *not relying on instantiation overflow*.
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Since 2006 [2], it is known that intuitionistic proposicional calculus **IPC** can be embedded into system $\mathbf{F_{at}}$—a restriction of Girard's polymorphic system **F** to atomic universal instantiations. Such embedding relies on the Russell–Prawitz's [4] translation of the connectives bottom and disjunction, $\perp := \forall X.X$ and $A \vee B := \forall X.\,((A \to X) \wedge (B \to X)) \to X$, and on the phenomenon of *instantiation overflow* [3]—the possibility of deriving in $\mathbf{F_{at}}$ the instantiation of these two universal formulas by any (not necessarily atomic) formula. In the present talk we show that there is an alternative (refined) embedding of **IPC** into $\mathbf{F_{at}}$, still based on the Russell–Prawitz's translation of connectives, but based on the admissability of disjunction and absurdity elimination rules, rather than instantiation overflow. Such alternative embedding works as well as the original embedding at the levels of provability and preservation of proof reduction (both embeddings preserve $\beta\eta$-conversions and map commuting conversions to β-equality) but the alternative embedding is more economical than the original one in terms of the size of the $\mathbf{F_{at}}$ proofs and the length of $\mathbf{F_{at}}$ simulations.

Acknowledgment. Details of this work can be found on [1].

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▶ MARTA FIORI CARONES, ALBERTO MARCONE, PAUL SHAFER, AND GIOVANNI SOLDÁ, *Reverse Mathematics of some principles related to partial orders*.
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In this talk, we will study (some variations of) the following theorem, due to Rival and Sands [3] in the context of Reverse Mathematics:

Theorem. (RS-po) *Let P be an infinite partial order of finite width k. Then there is an infinite chain C of P such that for every element $p \in P$, p is comparable with* 0 *or infinitely many elements of C.*

In particular, we show that ACA_0, the third of the Big Five subsystems of Z_2, is enough to prove RS-po, although no reversal is known to hold. An interesting result is obtained by fixing the width of the partial order P: if $k = 3$, we prove that the theorem is equivalent to ADS, a combinatorial principle introduced by Hirschfeldt and Shore in [2], and a widely studied element of the "zoo below ACA_0" (a very good presentation of which is given for instance in [1]). Notably, this version of the theorem appears to be the first natural mathematical statement proven to be equivalent to ADS.

Finally, some partial results on a stronger version of RS-po, where we require comparability with 0 or *cofinitely* many elements of C, will be presented.

[1] D. R. Hirschfeldt, ***Slicing the Truth: On the Computable and Reverse Mathematics of Combinatorial Principles***, World Scientific, 2015.

[2] D. R. Hirschfeldt and R. A. Shore, *Some principles weaker than Ramsey's theorem for pairs*. ***The Journal of Symbolic Logic***, vol. 72 (2007), no. 1, pp. 171–206.

[3] I. Rival and B. Sands, *On the adjacency of vertices to the vertices of an infinite subgraph*. ***Journal of the London Mathematica Society***, vol. s2-21 (1980), no. 3, pp. 393–400.

▶ MICHAEL STEPHEN FISKE, *Quantum random self-modifiable computation*.
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Among the fundamental questions in computer science, at least two have a deep impact on mathematics. What can computation compute? How many steps does a computation require to solve an instance of the 3-SAT problem? Our work addresses the first question, by introducing a new model called the *ex-machine* [3]. The ex-machine executes Turing machine instructions and two special types of instructions. *Quantum random instructions* are physically realizable with a quantum random number generator [4, 6]. *Meta instructions* can add new states and add new instructions to the ex-machine.

A countable set of ex-machines is constructed, each with a finite number of states and instructions; each ex-machine can compute a Turing incomputable language, whenever the quantum randomness measurements behave like unbiased Bernoulli trials. In 1936, Alan Turing posed the halting problem for Turing machines and proved that this problem is unsolvable for Turing machines. Consider an enumeration $\mathcal{E}_a(i) = (\mathfrak{M}_i, T_i)$ of all Turing machines $\mathfrak{M}_i$ and initial tapes T_i, each containing a finite number of nonblank symbols. Does there exist an ex-machine $\mathfrak{X}$ that has at least one evolutionary path $\mathfrak{X} \to \mathfrak{X}_1 \to \mathfrak{X}_2 \to \ldots \to \mathfrak{X}_m$, so at the mth stage ex-machine $\mathfrak{X}_m$ can correctly determine for $0 \leq i \leq m$ whether $\mathfrak{M}_i$'s execution on tape T_i eventually halts? We construct an ex-machine $\mathfrak{Q}(x)$ that has one such evolutionary halting path.

The existence of this path suggests that David Hilbert [5] may not have been misguided to propose that mathematicians search for finite methods to help construct mathematical proofs. Our refinement is that we cannot use a fixed computer program that behaves according to a fixed set of mechanical rules. We must pursue computational methods that exploit randomness and self-modification [1, 2] so that the complexity of the program can increase as it computes.

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[2] ———, *Quantum random active element machine*, ***Unconventional Computation and Natural Computation***, Lecture Notes in Computer Science, vol. 7956, Springer, 2013, pp. 252–254.

[3] ———, *Quantum random self-modifiable computation*, 2018, pp. 1–50, `https://arxiv.org/abs/1807.01369`.

[4] M. Herrero-Collantes and J. C. Garcia-Escartin, *Quantum random number generators*. ***Reviews of Modern Physics***, vol. 89, no. 1, p. 015004. APS, February 22, 2017.

[5] D. Hilbert, *Mathematische Probleme*. ***Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematische–Physikalische Klasse***, vol. 3 (1900), pp. 253–297.

[6] A. Kulikov, M. Jerger, A. Potočnik, A. Wallraff, and A. Fedorov, *Realization of a quantum random generator certified with the Kochen–Specker theorem*. ***Physical Review Letter***, vol. 119, p. 240501. December 11, 2017.

► MICHAL TOMASZ GODZISZEWSKI, DINO ROSSEGGER, AND LUCA SAN MAURO, *Quotient presentations of structures*.
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A *c.e. quotient presentation* of a structure $\mathcal{A} = \langle A; \{f_i\}_{i \in I}, \{R_j\}_{j \in J}\rangle$ consists of a structure $\mathcal{A}^* = \langle \mathbb{N}; \{f_i^*\}_{i \in I}, \{R_i^*\}_{j \in J}\rangle$ and a c.e. equivalence relation E (often called a *ceer*) such that the functions of $\mathcal{A}^*$ are uniformly computable, the relations of $\mathcal{A}^*$ are uniformly c.e., E is a congruence with respect to $\mathcal{A}^*$, and $\mathcal{A}^*/E \cong \mathcal{A}$. E *realizes* $\mathcal{A}$ if $(\mathcal{A}^*, E)$ is a c.e. quotient presentation of $\mathcal{A}$, for some $\mathcal{A}^*$; otherwise, E *omits* $\mathcal{A}$. Khoussainov and his collaborators (see, e.g., [2, 3]) investigated, for familiar classes of structures, which structures are realized by a given ceer E. We are interested in the reverse problem, that is, we study the structure of the following spectra.

Definition 1. The *spectrum of ceers* of a structure $\mathcal{A}$ is the following class of ceers

$$\mathrm{CeersSp}(\mathcal{A}) = \{E \in \mathit{Ceers}\colon E \text{ realizes } \mathcal{A}\}.$$

During the talk, we will discuss the main motivations for the project and we will demonstrate theorems relating the program to the study of some distinguished classes of equivalence relations, such as those considered in [1].

[1] U. Andrews and A. Sorbi, *Joins and meets in the structure of Ceers*. ***Computability***, forthcoming.

[2] E. Fokina, B. Khoussainov, P. Semukhin, and D. Turetsky, *Linear orders realised by c.e. equivalence relations*. ***The Journal of Symbolic Logic***, vol. 81 (2016), no. 2, pp. 463–482.

[3] A. Gavruskin, S. Jain, B. Khoussainov, and F. Stephan, *Graphs realised by r.e. equivalence relations*. ***Annals of Pure and Applied Logic***, vol. 165 (2014), no. 7, pp. 1263–1290.

▶ EVGENY GORDON, *On extension of Haar measure in σ-compact groups*.
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In the article [3] the model of ZFC, where every set of reals, definable by a sequence of ordinals is Lebesgue measurable was constructed under assumptions of existence of an inaccessible cardinal. On the base of this model the model of ZF+DC, in which every set of reals is Lebesgue measurable was presented. In [1], it was proved without the assumption of existence of inaccessible cardinal that the possibility to extend the Lebesgue measure to a nonregular σ-additive invariant measure defined on all sets of reals is consistent with ZF+DC. Later on Shelah proved that the assumption of existence of inaccessible cardinal cannot be removed from the Solovay's result [2]. In the talk we present the following theorem.

Theorem 1. *Let α be an arbitrary ordinal definable in ZF. Denote $Base(X, \beta)$ and $Ext(X, \beta)$ the statements*

1. *"X is a σ-compact group with the base of topology of cardinality β"*;
2. *"In a σ-compact group X the left Haar measure can be extended to a left invariant σ-additive measure defined on all subsets of X definable by a β-sequence of ordinals"*.

respectively. Then the following proposition is consistent with ZFC:

$$\forall X \forall \beta < \aleph_\alpha < |\mathbb{R}| \ \left(Base(X, \beta) \longrightarrow Ext(X, \beta)\right).$$

[1] G. Saks, *Measure-theoretical uniformity in recursion theory and set theory*. ***Transactions of the American Mathematical Society***, vol. 142 (1969), no. 2, pp. 381–420.

[2] S. Shelah, *Can you take Solovay's inaccessible away*? ***Israel Journal of Mathematics***, vol. 48 (1984), no. 1, pp. 1–47.

[3] R. Solovay, *A model of set theory in which every set of reals is Lebesgue measurable*. ***Annals of Mathematics***, vol. 92 (1970), no. 1, pp. 1–56.

▶ MATTIAS GRANBERG OLSSON AND GRAHAM LEIGH, *Partial conservativity of $\widehat{\mathrm{ID}}_1^{\mathrm{i}}$ over Heyting Arithmetic via realizability*.
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The result that intuitionistic $\widehat{\mathrm{ID}}_1$ ($\widehat{\mathrm{ID}}_1^{\mathrm{i}}$) is conservative over Heyting Arithmetic seems to have been proved only quite recently in a series of articles by Buchholz, Arai, and Rüede and Strahm [3, 1, 4, 2]. We present work in progress on a proposal for a hopefully novel proof of this result, or a substantial part of it, based on realizability and ideas from formal truth. The idea is to use Gödel's Diagonal Lemma to show that every axiom of some suitable subtheory of $\widehat{\mathrm{ID}}_1^{\mathrm{i}}$ (e.g., of fix-points only for strongly positive operators) is realizable, that realizability respects intuitionistic derivability and that realizability is disquotational for certain classes of formulae (e.g., almost negative formulae).

[1] T. Arai, *Some results on cut-elimination, provable well-orderings, induction and reflection*. ***Annals of Pure and Applied Logic***, vol. 95 (1998), no. 1–3, pp. 93–184.

[2] ———, *Quick cut-elimination for strictly positive cuts*. ***Annals of Pure and Applied Logic***, vol. 162 (2011), no. 10, pp. 807–815.

[3] W. Buchholz, *An intuitionistic fixed point theory*. ***Archive for Mathematical Logic***, vol. 37 (1997), no. 1, pp. 21–27.

[4] C. Rüede and T. Strahm, *Intuitionistic fixed point theories for strictly positive operators*. ***Mathematical Logic Quarterly***, vol. 48 (2002), no. 2, pp. 195–202.

► HAROLD HODES, *Ramified-types for states of affairs*.
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Assume that for any monadic predication P(u), which predicates the property being P of an object u, there is a unique state-of-affairs (which consists in u being P) which that predication represents; let *P(u)* be that state-of-affairs. I will give an argument that for every object u there are distinct properties being P and being Q such that *P(u)* = *Q(u)*. Use the following impredicative second-order comprehension principle: (G) some X every y (X(y) iff some Z (y = *Z(u)* and not Z(y))).

So far, no problem. But one might think that states-of-affairs have constituents, and that the following principle of constituency is true for any u and any property being P: (C) The constituents of *P(u)* are exactly u and being P.

By (C), the only constituents of *P(u)* are u and being P, and the only constituents of *Q(u)* are u and being Q, which entails that being P = being Q.

We could reject (C), at least in its full generality. Or we could say that (G) is defective. The former leads to a novel version of logical-atomist metaphysics. The latter points to a (to my knowledge) novel form of ramification.

► JOHN HOWE, *Ramsey degrees of structures with equivalence relations*.
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The Ramsey theory of homogeneous structures is an attempt to answer the question of how the infinite version Ramsey's theorem changes if instead of having ω as a pure set, we require more structure. This dates back to work of Galvin and Devlin on the rationals, with more recent results about graphs coming Sauer and Dobrinen. Both of these have used techniques involving tree Ramsey theorems, whereas Nguyen Van Thé's work about ultrametric spaces uses the classical Ramsey theorem. I will explain some recent work unifying these approaches and yielding results about the generic ordered equivalence relation.

► A. K. ISSAYEVA AND A. R. YESHKEYEV, *The principle of a "rheostat of atomicity" in the study of AAP models*.
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In this abstract, we want to share with the results concerning the study of countable algebraically prime and atomic models in the sense of studying inductive generally speaking incomplete theories.

Further we will have deal with countable language and some different subclasses of Jonsson theories.

Let AAP be a fixed semantic property, of the following properties combinations: A is atomicity, AP is algebraically primeness. Let AAP mean atomic, algebraically prime model.

Principle of "rheostat".

Let two countable models A_1, A_2 of some Jonsson theory T be given. Moreover, A_1 is an atomic model in the sense of [2], and X is $(\nabla_1, \nabla_2) - cl$-algebraically prime set of theory T and $cl(X) = A_2$. In the meaning of [3].

By the definition of (∇_1, ∇_2)-algebraic primeness of the set X, the model A_2 is in the same time existentially closed and algebraically prime. Thus, the model A_2 is isomorphically embedded in the model A_1. Since by condition the model A_1 is countably atomic, then according to the Vaught's theorem, A_1 is prime, that is, it is elementarily embedded in the model A_2. Thus, the models A_1, A_2 differ from each other only by the interior of the set X. This follows from the fact that any element of $a \in A_2 \backslash X$ implements some principle type, since $a \in cl(X)$. That is, all countable atomic models in the sense of [2] are isomorphic to each other, then by increasing X we find more elements that do not realize the principle type and, accordingly, $cl(X)$ is not an atomic model in the sense of [2]. Thus, the principle of rheostat is that, by increasing the power of the set X, we move away from the notion of atomicity in the sense of [2] and on the contrary, decreasing the power of the set X we move away from the notion of atomicity in the sense of [1].

In according above mentioned notions we have some numbers of theorems. Those results very close to investigation around atomicity and algebraically primeness in the frame of [1]. Nevertheless even if algebraically primeness is the same, but the combinations of AAP-atomicity differs from atomicity from [1].

[1] J. T. Baldwin and D. W. Kueker, *Algebraically prime models*. ***Annals of Mathematical Logic***, vol. 20 (1981), no. 3, pp. 289–330.

[2] R. Vaught, ***Denumerable Models of Complete Theories in Infinitistic Methode***, Pergamon, London, 1961, pp. 303–321.

[3] A. R. Yeshkeyev, A. K. Issayeva, and N. M. Mussina, *The atomic definable subsets of semantic model*. ***Bulletin of the Karaganda University-Mathematics***, vol. 94 (2019), no. 2, pp. 84–91.

► TATYANA IVANOVA AND TINKO TINCHEV, *First-order theory of lines in Euclidean plane*.
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The article [1] gives qualitative spatial reasoning in Euclidean plane based solely on lines. The relations of parallelism and convergence between lines are considered.

In this talk, we consider a continuation of [1] by adding a new predicate—perpendicularity. We introduce a first-order theory of lines in Euclidean plane with predicates parallelism, convergence and perpendicularity. The logic is complete with respect to the Euclidean plane, ω—categorical and not categorical in every uncountable cardinality. We prove that the membership problem of the logic is PSPACE-complete.

[1] P. Balbiani and T. Tinchev, *Line-based affine reasoning in Euclidean plane*. ***Journal of Applied Logic***, vol. 5 (2007), no. 3, pp. 421–434.

► JOOST J. JOOSTEN, *Hyperarithmetical Turing progressions*.
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Turing progressions arise by iteratedly adding consistency statements over a sound base theory. Schmerl employed Turing progressions over a weak base system in [2] to gauge the (consistency) strength of certain substantially stronger formal systems thus giving rise to ordinal analyses for these systems. Beklemishev showed in [1] how such analyses can be presented and in large part performed within polymodal provability logics. Beklemishev's method employed arithmetic consistency notions only. In this talk, we dwell on new techniques that have been developed to take this further to include hyperarithmetical consistency notions.

[1] L. D. Beklemishev, *Provability algebras and proof-theoretic ordinals, I*. ***Annals of Pure and Applied Logic***, vol. 128 (2004), no. 1–3, pp. 103–124.

[2] U. R. Schmerl, *A fine structure generated by reflection formulas over primitive recursive arithmetic*, ***Logic Colloquium '78 (Mons, 1978)***, Studies in Logic and the Foundations of Mathematics, vol. 97, North-Holland, 1979, pp. 335–350.

► HIROTAKA KIKYO, *On automorphism groups of Hrushovski's pseudoplanes in rational cases*.
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Hrushovski constructed pseudoplanes corresponding to irrational numbers which refute a conjecture by Lachlan [2]. Hrushovski's construction is valid for any real numbers α with

$0 < \alpha < 1$. The automorphism groups of the pseudoplanes corresponding to rational numbers α with $0 < \alpha < 1$ are simple groups.

[1] D. M. Evans, Z. Ghadernezhad, and K. Tent, *Simplicity of the automorphism groups of some Hrushovski constructions*. ***Annals of Pure and Applied Logic***, vol. 167 (2016), pp. 22–48.

[2] E. Hrushovski, *A stable $\aleph_0$-categorical pseudoplane*, unpublished notes, 1988.

[3] H. Kikyo, *Model completeness of generic graphs in rational cases*. ***Archive for Mathematical Logic***, vol. 57 (2018), no. 7–8, pp. 769–794.

[4] H. Kikyo and S. Okabe, *On Hrushovski's pseudoplanes*, ***Proceedings of the 14th and 15th Asian Logic Conference*** (B. Kim, J. Brendle, et al., editors), World Scientific, 2019, pp. 175–194.

▸ JULIA KNIGHT, ALEXANDRA SOSKOVA, AND STEFAN VATEV, *Effective coding and decoding structures*.

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Friedman and Stanley introduced Borel embeddings as a way of comparing classification problems for different classes of structures. A Borel embedding of a class K in a class K' represents a uniform procedure for coding structures from K in structures from K'. Many Borel embeddings are actually Turing computable.

When a structure $\mathcal{A}$ is coded in a structure $\mathcal{B}$, effective decoding is represented by a Medvedev reduction of $\mathcal{A}$ to $\mathcal{B}$. Harrison-Trainor, Melnikov, Miller, and Montalbán defined a notion of effective interpretation of $\mathcal{A}$ in $\mathcal{B}$ and proved that this is equivalent with the existing of computable functor, that is, a pair of Turing operators, one taking copies of $\mathcal{B}$ to copies of $\mathcal{A}$, and the other taking isomorphisms between copies of $\mathcal{B}$ to isomorphisms between the corresponding copies of $\mathcal{A}$. The first operator is a Medvedev reduction. For some Turing computable embeddings Φ, there are *uniform* formulas that effectively interpret the input structure in the output structure.

The class of undirected graphs and the class of linear orderings both lie on top under Turing computable embeddings. The standard Turing computable embeddings of directed graphs (or structures for an arbitrary computable relational language) in undirected graphs come with uniform effective interpretations. We give examples of graphs that are not Medvedev reducible to any linear ordering, or to the jump of any linear ordering. Any graph can be interpreted in some linear ordering using computable Σ_3 formulas. Friedman and Stanley gave a Turing computable embedding L of directed graphs in linear orderings. We show that there do not exist $L_{\omega_1\omega}$-formulas that uniformly interpret the input graph G in the output linear ordering $L(G)$.

▸ THOMAS G. KUCERA AND MARCOS MAZARI-ARMIDA, *On universal modules with pure embeddings*.

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This article arose out of the realization of the second author that some notions of the theory of abstract elementary classes can be used to generalize a result of Shelah [2, 1.2] concerning the existence of universal reduced torsion-free abelian groups with respect to pure embeddings. The contribution of the first author was limited to helping him expand and extend the results to theories of modules.

We show that certain classes of modules have universal models with respect to pure embeddings.

THEOREM 1. *Let R be a ring, T a first-order theory with an infinite model extending the theory of R-modules and $\mathbf{K}^T = (Mod(T), \leq_{pp})$ (where $\leq_{pp}$ stands for pure submodule). Assume $\mathbf{K}^T$ has joint embedding and amalgamation.*

If $\lambda^{|T|} = \lambda$ or $\forall \mu < \lambda(\mu^{|T|} < \lambda)$, then $\mathbf{K}^T$ has a universal model of cardinality λ.

We begin the study of limit models for classes of R-modules with joint embedding and amalgamation. As a by-product of this study, we characterize limit models of countable cofinality in the class of torsion-free abelian groups with pure embeddings, answering Question 4.25 of [1].

THEOREM 2. *If G is a (λ, ω)-limit model in the class of torsion-free abelian groups with pure embeddings, then $G \cong \mathbb{Q}^{(\lambda)} \oplus \prod_p \overline{\mathbb{Z}_{(p)}^{(\lambda)}}^{(\aleph_0)}$.*

[1] M. MAZARI-ARMIDA, *Algebraic description of limit models in classes of abelian groups*, preprint, https://arxiv.org/abs/1810.02203.

[2] S. SHELAH, *Universal structures*. ***Notre Dame Journal of Formal Logic***, vol. 58 (2017), no. 2, pp. 159–177.

▶ TAISHI KURAHASHI, *Derivability conditions and the second incompleteness theorem*.
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Let T be any recursively axiomatized consistent extension of Peano arithmetic. In his famous article, Gödel showed that the consistency statement $\mathrm{Con}_T \equiv \exists x(\mathsf{Fml}(x) \wedge \neg \mathrm{Pr}_T(x))$ cannot be proved in T. In the second volume of *Grundlagen der Mathematik*, Hilbert and Bernays proposed a set of conditions for provability predicates which is sufficient for a version of the second incompleteness theorem. That is, if $\mathrm{Pr}_T(x)$ is a Σ_1 provability predicate satisfying their conditions, then $\mathrm{Con}_T^0 \equiv \forall x(\mathsf{Fml}(x) \wedge \mathrm{Pr}_T(x) \to \neg \mathrm{Pr}_T(\neg x))$ cannot be proved in T. Löb [4] found another set of conditions, and proved the so-called Löb's theorem under his conditions. Löb's theorem immediately implies that $\mathrm{Con}_T^1 \equiv \mathrm{Pr}_T(\ulcorner 0 \neq 0 \urcorner)$ cannot be proved in T. Notice that for provability predicates, Con_T^0 implies Con_T^1, and Con_T^1 implies Con_T.

Related to derivability conditions and the second incompleteness theorem, we proved the following results.

1. There are new sets of derivability conditions which are sufficient for unprovability of Con_T^0.
2. If a Σ_1 provability predicate $\mathrm{Pr}_T(x)$ satisfies the following condition $\boldsymbol{B_2^U}$, then $\mathrm{Pr}_T(x)$ satisfies provable Σ_1-completeness.

$$\boldsymbol{B_2^U}: \text{If } T \vdash \varphi(\vec{x}) \to \psi(\vec{x}), \text{ then } T \vdash \mathrm{Pr}_T(\ulcorner \varphi(\dot{\vec{x}}) \urcorner) \to \mathrm{Pr}_T(\ulcorner \psi(\dot{\vec{x}}) \urcorner).$$

   This is an improvement of Buchholz's observation [1].
3. Hilbert and Bernays' conditions and Löb's conditions are incomparable.
4. Both of Hilbert and Bernays' conditions and the global versions of Löb's conditions are not sufficient for $T \nvdash \mathrm{Con}_T$. This shows that both of Hilbert-Bernays' conditions and Löb's conditions do not accomplish Gödel's original statement of the second incompleteness theorem.

[1] W. BUCHHOLZ, *Mathematische Logik II*, 1993, `http://www.mathematik.uni-muenchen.de/~buchholz/articles/LogikII.ps`.

[2] T. KURAHASHI, *A note on derivability conditions*, `arXiv:1902.00895`.

[3] ———, *Rosser provability and the second incompleteness theorem*, `arXiv:1902.06863`.

[4] M. H. LÖB, *Solution of a problem of Leon Henkin*. ***The Journal of Symbolic Logic***, vol. 20 (1955), no. 2, pp. 115–118.

▶ SATORU KURODA, *On Takeuti-Yasumoto forcing*.
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In late 1996, G. Takeuti and M. Yasumoto [1] published a article on applications of forcing method for nonstandard models of bounded arithmetic.

In this talk, we will give a reformulation of their forcing construction in terms of two-sort bounded arithmetic. In particular, we will construct Boolean algebras on which generic extensions are models for theories for subclasses of PTIME such as NC^1 or NL. For instance, let $\mathbb{B}$ be the Boolean algebra whose underlying set consists of Boolean formulas over n inputs where n is a fixed nonstandard number. Then a generic subset of $\mathbb{B}$ constitutes a generic extension which is a model of $\mathbf{VNC}^1$.

It turns out that such generic extensions have close connections with separation problems of complexity classes in the ground model. Namely let $\mathfrak{M} \models \mathbf{V}^1$ be a countable nonstandard model which is not closed under exponentiation. Then we can show that $\mathfrak{M} \models (NC^1 = P)$ if and only if any generic extension based on Boolean algebra for NC^1 is a model of $\mathbf{VP}$.

We will also discuss the problem of relating propositional provability in the ground model and the generic extension.

[1] G. Takeuti and M. Yasumoto, ***Forcing on Bounded Arithmetic***, Lecture Notes in Logic, vol. 6, Cambridge University Press, 1996, pp. 120–138.

▸ MICHAEL LIEBERMAN, JIŘÍ ROSICKÝ, AND SEBASTIEN VASEY, *Weak factorization systems and stable independence*.
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We discuss recent joint work with Rosický and Vasey, [2], which reveals surprising connections between model-theoretic independence notions and the behavior of *weak factorization systems*, which play an important role in the analysis of model categories and in homological algebra. In essence, given a reasonable category $\mathcal{K}$ and family of maps $\mathcal{M}$, the category $\mathcal{K}_{\mathcal{M}}$ obtained by restricting to the morphisms in $\mathcal{M}$ has a stable independence notion just in case $\mathcal{M}$ forms the left half of a *cofibrantly generated* weak factorization system, that is, one generated by pushouts and transfinite compositions from a set—rather than a class—of basic maps. We sketch the argument, recalling the category-theoretic generalization of stable nonforking independence from [2], as well as the necessary terminology involving weak factorization systems.

As a particular example, we specialize to the case $\mathcal{K} = R$-Mod and $\mathcal{M}$ a class of homomorphisms with kernels in a fixed subcategory: this generalizes the (abstract elementary) classes of modules ${}^{\perp}N$ considered by Baldwin–Eklof–Trlifaj, [3], and answers a number of questions from their article. In particular, we prove that this class is tame and stable whenever it is an AEC.

[1] J. Baldwin, P. Eklof, and J. Trlifaj, *${}^{\perp}N$ as an abstract elementary class*. ***Annals of Pure and Applied Logic***, vol. 149 (2007), no. 1–3, pp. 25–39.

[2] M. Lieberman, J. Rosický, and S. Vasey, *Weak factorization systems and stable independence*, submitted, arXiv:1904.05691v2.

[3] ———, *Forking independence from the categorical point of view*. ***Advances in Mathematics***, vol. 346 (2019), pp. 719–772.

▸ ROBERT S. LUBARSKY, *Feedback hyperjump*.
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Under feedback computability, the halting problem relative to the halting problem is the halting problem: $X = X'$. Most, if not all, notions of computation that allow for an oracle have a feedback version. The ones that have been explored so far are Turing computability, primitive recursion, and infinite time Turing machines. This talk will include an introduction to feedback, and the current state of knowledge about feedback hyperjump $(X = \mathcal{O}^X)$.

[1] N. ACKERMAN, C. FREER, and R. LUBARSKY, *Feedback computability on Cantor space*, ***Logical Methods in Computer Science***, LICS 2015. Special Issue, to appear; also available at `http://math.fau.edu/lubarsky/pubs.html`.

[2] ———, *An introduction to feedback Turing computability*, ***Annals of Pure and Applied Logic***, LFCS 2016. Special Issue, submitted; also available at `http://math.fau.edu/lubarsky/pubs.html`.

[3] R. LUBARSKY, *ITTMs with feedback*, ***Ways of Proof Theory*** (R. Schindler, editor), Ontos, Eichenweg 25, Ortenberg 63683, Germany, 2010, pp. 341–354. `http://www.ontos-verlag.de`, `http://www.ontoslink.com/`.

[4] ———, *Parallel feedback Turing computability*, ***Proceedings of LFCS 2016*** (Artemov and Nerode, editors), Lecture Notes in Computer Science, vol. 9537.

▸ PATRICK LUTZ AND JAMES WALSH, *Descending sequences of hyperdegrees and the second incompleteness theorem*.
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It follows from classical results due to Spector that there is no sequence of reals $A_0, A_1, A_2, \dots$ such that for each n, $A_n \geq_H \mathcal{O}^{A_{n+1}}$. We will give a new proof of this result using the second incompleteness theorem. We will then mention how this fact can be used to give an alternative proof of a result of Simpson and Mummert on a semantic version of the second incompleteness theorem for β_n models. Both of these results seem to suggest a more general connection between well-foundedness of certain partial orders and the second incompleteness theorem. We will mention several other examples of this connection.

▸ ALICE MEDVEDEV AND ALEXANDER VAN ABEL, *The Feferman–Vaught Theorem and products of finite fields*.
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We prove that in a product ring of finite fields, the definable subsets are boolean combinations of $\exists\forall\exists$-definable sets. This follows from the Feferman–Vaught Theorem on definability in product structures [1], and Kiefe's quantifier reduction result for finite fields [2]. We obtain via our proof that products of integral domains have the maximum amount of definable subsets allowed by the Feferman–Vaught theorem.

[1] S. FEFERMAN and R. VAUGHT, *The first order properties of products of algebraic systems*. ***Fundamenta Mathematicae***, vol. 47 (1959), no. 1, pp. 57–103.

[2] C. KIEFE, *Sets definable over finite fields*: *Their zeta-functions*. ***Transactions of the American Mathematical Society***, vol. 223 (1976), pp. 45–59.

▸ JOSÉ M. MÉNDEZ, GEMMA ROBLES, AND FRANCISCO SALTO, *Falsity constants for two independent families of quasi-Boolean logics*.
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In [1], two families of quasi-Boolean logics are defined. One of them is intuitionistic in character; the other one, dual intuitionistic in nature. Both families are determined by a Routley-Meyer ternary relational semantics, negation being interpreted by the "Routley operator" or "Routley star". The aim of this article is to reconsider the two aforementioned families from the point of view of the same semantics except that now negation will be interpreted by means of two different types of falsity constants following the techniques and strategies discussed in [2]. We will compare the results obtained with those recorded in [1].

Acknowledgment. Work supported by research project FFI2017-82878-P, financed by the Spanish Ministry of Economy, Industry and Competitiveness.

[1] J. M. Méndez, G. Robles, and F. Salto, *Basic quasi-Boolean extensions of relevant logics* (*abstract*), this Bulletin, Contributed talk presented at the ASL European Summer Meeting (Logic Colloquium 2018), Udine, Italy, 23–28 July, 2018, forthcoming.

[2] G. Robles and J. M. Méndez, ***Routley–Meyer Ternary Relational Semantics for Intuitionistic-type Negations***, Elsevier, 2018.

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In stable theories it is possible to associate to sufficiently big models a certain monoid obtained by quotienting the semigroup of types with tensor product by a relation called "domination-equivalence". This equivalence relation was generalised to arbitrary theories in [1], where it was studied in the case of the theory of algebraically closed valued fields and it was shown that every global invariant type is domination-equivalent to a product of types concentrating in the residue field or in the value group. Unfortunately, domination-equivalence is not always a congruence with respect to the product of invariant types, as shown in [2]. The aim of this talk is to present an instance of this incompatibility, along with a first development of the general theory of this interaction.

[1] D. Haskell, E. Hrushovski, and D. Macpherson, ***Stable Domination and Independence in Algebraically Closed Valued Fields***, Lecture Notes in Logic, Cambridge University Press, 2008.

[2] R. Mennuni, *Product of invariant types modulo domination-equivalence*. ***Archive for Mathematical Logic***, accepted.

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Renaissance geometry refers directly or indirectly to Euclidean geometry. Fibonacci's *Practica geometriae* written in 1220 contains a large collection of geometry problems arranged into eight chapters with theorems based on Euclid's *Elements*. Piero della Francesca in his treatise solely devoted to the subject of perspective *De Prospectiva Pingendi*, written possibly by about 1474, refers to many Euclid's theorems. For instance in Proposition 1.13, which is known as the first new European theorem in geometry after Fibonacci, the proof refers to the similarity of the triangles. In *Elements* discussion of these issues is included in the Book VI, Proposition 4 to 8. In turn to determine the height of a man one can use the rectangle. The method refers to Euclidean Proposition 16, Book 4, which involves constructing a fifteen-sided figure, equilateral and equiangular. What, however, is the most interesting these and other propositions can be used in the interpretation of the paintings of Piero della Francesca. Luca Pacioli, the pupil of Piero, in his *De divina proportione* moved the mathematical and artistic problems of proportion, especially the mathematics of the golden ratio and its application in architecture.

The purpose of the study is to describe and compare Renaissance geometry in combination with Euclidean one.In the Renaissance the mathematical sciences were in the center of attention and there was a close union between them and the fine arts.

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Moral conflicts are special kind of situations that arise as a reaction to dealing with the conflicting obligations. The resolution of moral conflicts has been studied extensively within the area of moral reasoning whereas representation within the framework of Deontic logic. Despite of moral conflicts are very much part of our linguistic discourse and our tolerance towards them is frequent phenomena, yet the core principles of Standard Deontic Logic fail to capture the intuitive notion of moral conflicts in a satisfactory manner. This poses a major challenge in handling moral conflicts. We argue that situations involving moral conflicts mainly concerned with tolerating inconsistencies, and we assume that best known framework for dealing with moral conflicts are the deontic logics extended with the paraconsistent logic. In paraconsistent logics, a conflict can be represented, operated, isolated, without invalidating the inference rules. I examine three prominent paraconsistent logics; Grahm Priest's logic LP, the logic RM of the school of relevance logic and the Da Costa's logics Cn based on the three valued logic. We emphasize on Deontic paraconsistent logics based on Priest's paraconsisntent logic. I illustrate my work with a classic example from famous Indian epic 'Mahabharata' where the protagonist Arjuna faces moral conflict in the battlefield of Kurukshetra. The inquiry is to find an adequate set of principles to accommodate Arjuna's moral conflict in paraconsistent deontic logics. Meanwhile, it is also interesting to relate Krishna's arguments for resolving Arjuna's conflict to paraconsistent approach of conflict tolerance.

[1] N. C. A. Da Costa and W. A. Carnielli, *On paraconsistent deontic logic*. ***Philosophia***, vol. 16 (1986), no. 3, pp. 293–305.

[2] D. Gabbay, J. Horty, X. Parent, R. van der Meyden, and L. van der Torre, ***Handbook of Deontic Logic and Normative Systems***, College Publication, 2013.

▶ ANDREY MOROZOV AND JAMALBEK TUSSUPOV, *On minimal elements in the* Δ-*reducibility on families of predicates*.
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Fix some countable set U. By *predicate* here we mean an arbitrary subset of an arbitrary finite Cartesian power of U. We study two kinds of reducibilities on finite families of predicates.

We say that a predicate R is Δ-*definable over the predicates* $P_1, \dots, P_k$ if R itself and its complement can be defined in the structure $\langle U; P_1, \dots, P_k\rangle$ by means of ∃-formulas with parameters.

Let $S_0 = \{P_0, \dots, P_{k-1}\}$ and S_1 be two finite families of predicates. We say that S_0 *is* Δ-*definable in* S_1, if all the predicates in S_0 are Δ-definable in S_1 and we denote this fact as $S_0 \leqslant^0_\Delta S_1$. If $S_0 \leqslant^0_\Delta S_1$ and $S_1 \leqslant^0_\Delta S_0$ then we denote this fact as $S_0 \equiv^0_\Delta S_1$. The relation $\leqslant^0_\Delta$ is a preordering, $\equiv^0_\Delta$ is an equivalence and the quotient $\leqslant^0_\Delta/_{\equiv^0_\Delta}$ defines an upper semilattice in which the least upper bound of elements $S_0/_{\equiv^0_\Delta}$ and $S_1/_{\equiv^0_\Delta}$ equals to $(S_0 \cup S_1)/_{\equiv^0_\Delta}$ and $\perp^0_\Delta = \varnothing/_{\equiv^0_\Delta}$ is the smallest element. Denote this semilattice by D^0_Δ.

If we consider families of predicates up to isomorphism, we arrive at the notion of Δ-*reducibility on families of predicates*. We say that a finite family of predicates S_0 Δ-*reduces to a finite family* S_1 (and denote this as $S_0 \leqslant_\Delta S_1$), if there exists a finite family of predicates S' such that $S'_0 \leqslant^0_\Delta S_1$ and S'_0 is a conjugate of S_0 by means of some permutation on U.

If $S_0 \leqslant_\Delta S_1$ and $S_1 \leqslant_\Delta S_0$ then we denote this fact as $S_0 \equiv_\Delta S_1$. The quotient $\leqslant_\Delta/_{\equiv_\Delta}$ defines a structure D_Δ, which is a partial order with smallest element $\perp_\Delta = \varnothing/_{\equiv_\Delta}$.

THEOREM 1.

1. *The structure D_Δ fails to be an upper semilattice.*
2. *The families consisting of unary predicates define in D_Δ an ideal of order type ω.*

THEOREM 2. *Each of the structures $D^0_\Delta \setminus \{\perp^0_\Delta\}$ and $D_\Delta \setminus \{\perp_\Delta\}$ contains 2^ω minimal elements.*

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It seems at least unlikely that there is an natural way to extend the $\{0, 1\}$-interpretations of PL to arbitrary values from the unit interval $[0, 1] = \{x \in \mathrm{IR} : 0 \leq x \leq 1\}$.

0. For technical reasons, we use the following PL-language: $p_1, p_2, p_3, \ldots, \neg\phi, \bigwedge\Phi, \bigvee\Psi$, where Φ, Ψ stand for *finite* sets of formulæ previously built. For instance, $\phi \wedge \psi := \bigwedge\{\phi, \psi\}$. *apv*$(\phi)$ be the set of atomic propositional variables p occurring in ϕ.

Formulæ are binarily interpreted in well-known way, e. g. $V(p) \in \{0, 1\}$, $V'(\neg\phi) = 1$ iff $V'(\phi) = 0$, $V'(\bigwedge\Phi) = 1$ iff $V'(\phi) = 1$ for all $\phi \in \Phi$. $V'(\bigvee\emptyset) = 0$. $\models \phi$:iff $V'(\phi) = 1$ for all V, whence $\models \phi \leftrightarrow \psi$ iff $V'(\phi) = V'(\psi)$.

I. Let $\mathrm{apv}(\phi) = \{p^1, \ldots, p^n\}$. Call $\kappa := \bigwedge\Lambda$ *fundamental* iff either $p^k \in \Lambda$ or $\neg\, p^k \in \Lambda$ for every $1 \leq k \leq n$. $\kappa_1 \perp \kappa_2$ if $\Lambda_1 \neq \Lambda_2$. There is an unique set K of fundamental κ such that $\models \phi \leftrightarrow \bigvee K$, where $\delta := \bigvee K$ corresponds to (full) DNF.

Now let $W(p) \in [0, 1]$ be any *multi-valued assignment*. The *MVI* or *Buchholz valuation* W' is constructed as follows:
$W'(p^k) := W(p^k)$,
$W'(\neg p^k) := 1 - W'(p^k)$,
$W'(\kappa) := \prod_{\lambda\in\Lambda} W'(\lambda)$,
$W'(\delta) := \sum_{\kappa\in K} W'(\kappa)$,
$W'(\phi) := W'(\delta)$;
revealing the somehow evident principles used.

Let $W_{0.5} := \frac{1}{2}$. $W'_{0.5} = \pi_{\mathrm{PL}}$ has been *paradigm* for MVI, whereby *PL-probability* $\pi_{\mathrm{PL}}(\phi)$ equals the number of rows with value 1 in the (binary) truth table of ϕ divided by 2^n—originating with the *Tractatus*, probably.

II. Except for $\pi(\bigwedge\Lambda) = \prod_{\lambda\in\Lambda} \pi(\lambda)$, the principles used to define W' are all properties of probability functions π in the sense of probability logic—maybe of many-valued logic at all—, and we showed that *the literal-independent* π can be identified with our W'.

However, the nearness to stochastics is deceptive: Consider, e. g., coin toss, where $\pi(\text{heads} \wedge \text{tails}) \neq \pi(\text{heads}) \cdot \pi(\text{tails})$.

III. What have we found? The status of literal-independency has remained unclear.

Acknowledgments. After a joint quest, WILFRIED BUCHHOLZ had solved the problem technically.—Preliminary versions were presented at ASL-APA and UniLog 2018. Thanks to many participants, Walter Carnielli, Luis Estrada-González, Peter Maier-Borst, Schafag Kerimova, and Andreas Haltenhoff.

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The Cut Rule as a structural rule used in sequent calculi can be seen in the context of justification of deduction as a recognition of the possibility of indirect proofs for a sentence having logical constant(s). The demand for Cut Elimination Theorem for a calculus having logical constants can be seen, from this perspective, as the demand for showing that if there is an indirect proof for such a sentence then there is a direct proof for it as well. It can be shown that a calculus which has Cut Elimination Theorem for it satisfies Belnap's ("Tonk, Plonk and Plink", 1962) condition of being a conservative extension of the source calculus (S: deducibility as such) comprising of only structural rules including Cut, and the Axiom of

Identity. Belnap held that an extended calculus having logical constants should also satisfy the condition of uniqueness.

Restall ("General Existence 1 : Quantification and Free Logic", 2019) takes sequents as proof-theoretic representations of 'clash' between assertions and denials of formulae. Restall shows that his Defining Rules for the classical first order logical constants make way for, not only a conservative extension of S into Classical First Order Predicate (Free) Logic, but also for an uniquely defining extension as well. Restall shows how the usual left/right sequent rules for the constants can be restored from the Defining Rules. For such a restoration Axiom of Identity and the Cut rule become necessary for him. This article observes that this necessary use of Cut here importantly shows that what is achieved by a Cut Elimination Theorem for a usual calculus (as discussed above) is achieved by Restall's calculus with Defining Rules too, but of course without demanding that Cut be eliminable.

Some ramifications of this feature of Restall's calculus are explored.

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As for *cut elimination* (see [3, p. 93]), we say that the *subterm property* holds for a sequent calculus $\boldsymbol{S}$ if there is a "nontrivial" algorithm for transforming a derivation in $\boldsymbol{S}$ of a sequent S into a derivation of S in the same system, that contains only terms occurring in S. We show that the subterm property holds for the following purely equality calculi based on the structural rules:

1. $*\mathbf{EQ}_N$ (N for "natural"), which has the reflexivity axioms $\Rightarrow t = t$ and the multiple congruence rule

$$\frac{\Gamma \Rightarrow r_1 = s_1 \qquad \dots \qquad \Gamma \Rightarrow r_n = s_n \qquad \Gamma \Rightarrow F[v_1/r_1, \dots, v_n/r_n]}{\Gamma \Rightarrow F[v_1/s_1, \dots, v_n/s_n]}$$

2. $*\mathbf{EQ}_B$ (B for "Birkhoff"), which has the reflexivity axioms and the rules:

$$\frac{\Gamma \Rightarrow r = s}{\Gamma \Rightarrow s = r} \qquad\qquad \frac{\Gamma \Rightarrow r = s \qquad \Gamma \Rightarrow s = t}{\Gamma \Rightarrow r = t}$$

$$\frac{\Gamma \Rightarrow r_1 = s_1 \qquad \dots \qquad \Gamma \Rightarrow r_n = s_n}{\Gamma, P[v_1/r_1, \dots, v_n/r_n] \Rightarrow P[v_1/s_1, \dots, v_n/s_n]}$$

$$\frac{\Gamma \Rightarrow r_1 = s_1 \qquad \dots \qquad \Gamma \Rightarrow r_n = s_n}{\Gamma \Rightarrow t[v_1/r_1, \dots, v_n/r_n] = t[v_1/s_1, \dots, v_n/s_n]}$$

3. $*\mathbf{EQ}$, which has the reflexivity axioms and the rules

$$\frac{\Gamma \Rightarrow F[v_1/r_1, \dots, v_n/r_n]}{r_1 = s_1, \dots, r_n = s_n, \Gamma \Rightarrow F[v_1/s_1, \dots, v_n/s_n]}$$

$$\frac{\Gamma \Rightarrow F[v_1/r_1, \dots, v_n/r_n]}{s_1 = r_1, \dots, s_n = r_n, \Gamma \Rightarrow F[v_1/s_1, \dots, v_n/s_n]}$$

where Γ is a finite multiset of formulae, F is a formula, P is an atomic formula different from an equality, r,s, t, the r_i's and s_i's are terms and the v_i's are variable of a first order language and $E[v_i/t_1, \dots, v_i/t_n]$ is used to denote the result of the simultaneous replacement of the free variables $v_1, \dots, v_n$ by the terms $t_1, \dots, t_n$ in the formula or term E.

Moreover, for $*\mathbf{EQ}_N$ and $*\mathbf{EQ}_B$ cut elimination and the subterm property hold simultaneously, namely a derivation in any of such systems of a sequent S can be transformed into a cut-free derivation of S in the same system, containing only terms occurring in S. Although cut elimination holds also for $*\mathbf{EQ}$, it does not hold simultaneously with the subterm property.

[1] F. Parlamento and F. Previale, *Absorbing the structural rules in the sequent calculus with additional atomic rules*, *Logic Colloquium 2018*, this Bulletin, vol. 25 (2019), no. 2, pp. 260–261.

[2] ———, *The cut elimination and nonlengthening property for the sequent calculus with equality*, *Logic Colloquium 2016*, this Bulletin, vol. 23 (2017), no. 2, pp. 251–252.

[3] A. S. Troelstra and H. Schwichtenberg, ***Basic Proof Theory***, second ed., Cambridge University Press, Cambridge, 2000.

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Let A be an arbitrary model of countable language. $JSp(A) = \{T/T$ is Jonsson theory in this language and $A \in ModT\}$ and $JSp(A)$ is said to be the Jonsson spectrum of the model A.

Definition 1. We say that the Jonsson theory T_1 is cosemantic to the Jonsson theory T_2 $(T_1 \bowtie T_2)$ if $C_{T_1} = C_{T_2}$, where C_{T_i} are semantic model of T_i, $i = 1, 2$.

The relation of cosemanticness on a set of Jonsson theories is an equivalence relation. Then $JSp(A)/\bowtie$ is the factor set of the Jonsson spectrum of the model A with respect to $\bowtie$.

Let us define the essence of the symbol $\boxdot$ of the operation for algebraic construction of models, which will play an important role in the definition of hybrids. Let symbol $\boxdot \in \{\cup, \cap, \times, +, \oplus, \prod_F, \prod_U\}$, where $\cup$-union, $\cap$-intersection, $\times$-Cartesian product, $+$-sum and $\oplus$-direct sum, $\prod_F$-filtered product and $\prod_U$-ultraproduct.

Definition 2. A hybrid of classes $[T]_1, [T]_2$ is the class $[T]_i \in JSp(A)/\bowtie$ if $Th_{\forall\exists}(C_1 \boxdot C_2) \in [T]_i$, we denote such hybrid as $H([T]_1, [T]_2)$.

Note the following fact:

Fact 1. *For the theory* $H([T]_1, [T]_2)$ *in order to be Jonsson enough to be that* $(C_1 \boxdot C_2) \in E_{[T]_i}$, *where* $[T]_i \in JSp(A)/\bowtie$.

Finally, the main results are the following theorem.

Theorem 3. *Let* $[T]_1, [T]_2$ *be perfect convex existentially prime complete for* $\forall\exists$*-sentences classes from* $JSp(A)/\bowtie$. X_i *are* $\forall\exists$*-dcl-sets in the class* $[T]_i$, $i \in \{1, 2\}$, *that is,* $X_i \subseteq C_i$, *where* $M_i = dcl(X_i) \in E_{[T]_i}$, $T_i = Th_{\forall\exists}(M_i)$ *are also perfect convex existentially prime complete for* $\forall\exists$*-sentences Jonsson theories. Then, if their hybrid* $H([T]_1, [T]_2)$ *is a model consistent with* $[T]_i$, *then* $H([T]_1, [T]_2)$ *is a perfect class from* $JSp(A)/\bowtie$ *for* $i = 1, 2$.

Theorem 4. *Let* $[T]_1, [T]_2$ *satisfy the conditions of Theorem* 1 *and* $[T]_1, [T]_2$ *be* ω*-categorical. Then their hybrid* $H([T]_1, [T]_2)$ *is also a* ω*-categorical class from* $JSp(A)/\bowtie$.

All concepts that are not defined in this abstract can be extracted from [1, 2].

[1] M. T. Kassymetova and A. R. Yeshkeyev, ***Jonsson Theories and their Classes of Models***, KSU, Karaganda, [in Russian], 2016.

[2] N. M. Mussina and A. R. Yeshkeyev, *Properties of hybrids of Jonsson theories*. ***Bulletin of the Karaganda University. Seria of "Mathematics"***, vol. 92 (2018), no. 4, pp. 99–104.

▸ MANAT MUSTAFA AND SERGEY OSPICHEV, *About Rogers semilattices of finite families in Ershov hierarchy*.
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There is a well-known result, that any finite family of c.e. sets has computable principal numbering[3]. In [1], K. Abeshev shows that there is a finite family of sets in Ershov hierarchy without Σ_2^{-1}-computable principal numbering. With the help of Γ-operator in [2], above result can be generalized to any level (finite and successor ordinals) of Ershov hierarchy. Here we concentrate our interest to different types of Σ_2^{-1}-computable numberings of finite families of Σ_2^{-1}-sets and c.e.-sets. The main result is:

THEOREM. *Let $\mathcal{S} = \{A, B\}$ be any family with A, B are c.e. sets with $A \subseteq B$ but $A \setminus B$ is not c.e., then the Rogers semilattice $\mathcal{R}_2^{-1}(\mathcal{S})$ is isomorphic to family L_0^m of all m-degrees of c.e. sets.*

COROLLARY. *Any Σ_2^{-1}-computable numbering of $\mathcal{S}$ is equivalent to some computable numbering of $\mathcal{S}$.*

Acknowledgment. Second author was supported by RFBR according to the research project 17-01-00247.

[1] K. ABESHEV, *On the existence of universal numberings for finite families of d.c.e. sets*. ***Mathematical Logic Quarterly***, vol. 60 (2014), no. 3, pp. 161–167.

[2] I. HERBERT, S. JAIN, S. LEMPP, M. MUSTAFA, and F. STEPHAN, *Reductions between types of numberings*, preprint, `www.math.wisc.edu/~lempp/papers/redandnumb.pdf`.

[3] A. H. LACHLAN, *Standard classes of recursively enumerable sets*. ***Zeitschrift für mathematische Logik und Grundlagen der Mathematik***, vol. 10 (1964), no. 2–3, pp. 23–42.

▸ RICARDO NICOLÁS-FRANCISCO AND LUIS ESTRADA-GONZÁLEZ, *Negation can be just what it has to*.
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According to Jc Beall [1], there is no logical negation because a logical negation must be either exclusive or exhaustive, but there are no logical reasons that force negation in either way in the correct logic—that, for reasons that we cannot reproduce here, has to be subclassical for Beall (see [2]). In this article, we provide some counterarguments to Beall. In particular, we probe characterizations of negation that do not involve the need for exhaustion or exclusion, for example, the flip-flop character of negation as present in Beall's preferred subclassical logic: FDE.

[1] J. BEALL, *There is no logical negation*: *True, false, both and neither*. ***Australasian Journal of Logic***, vol. 14 (2017), no. 1 pp. 1–29.

[2] ———, *The simple argument for subclassical logic*. ***Philosophical Issues***, vol. 28 (2018), no. 1, pp. 30–54.

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We continue to study families of theories of abelian groups [3] characterizing e-minimal subfamilies [4] for finite abelian groups by Szmielew invariants $\alpha_{p,n}$, β_p, γ_p, ε [2, 1], where p are prime numbers, $n \in \omega \setminus \{0\}$, as well as describing possibilities for the rank $\mathrm{RS}(\cdot)$ [4].

We denote by $\mathcal{T}_{A,\mathrm{fin}}$ the family of all theories of finite abelian groups.

THEOREM 1. *For any infinite family $\mathcal{T} \subseteq \mathcal{T}_{A,\mathrm{fin}}$ the following conditions are equivalent*: (1) *$\mathcal{T}$ is e-minimal*; (2) $\dim(\mathcal{T}) = 1$, *that is, $\mathcal{T}$ does not have independent limit values for Szmielew invariants*; (3) *for any upper bound $\alpha_{p,n} \geq m$ or lower bound $\alpha_{p,n} \leq m$, for $m \in \omega$, there are finitely many theories in $\mathcal{T}$ satisfying this bound*; *having finitely many theories with $\alpha_{p,n} \geq m$, there are infinitely many theories in $\mathcal{T}$ with a fixed value $\alpha_{p,n} < m$.*

THEOREM 2. *Let α be a countable ordinal, $n \in \omega \setminus \{0\}$. Then there is a subfamily $\mathcal{T} \subset \mathcal{T}_{A,\mathrm{fin}}$ such that* $\mathrm{RS}(\mathcal{T}) = \alpha$ *and* $\mathrm{ds}(\mathcal{T}) = n$.

The families $\mathcal{T}$ for the proof of Theorem 2 have closures $\mathrm{Cl}_E(\mathcal{T})$ inside $\mathcal{T}_{A,\mathrm{fin}} \cup \mathcal{T}_{A,\mathrm{pf}}$, where $\mathcal{T}_{A,\mathrm{pf}}$ is the set of theories of pseudofinite abelian groups, and these closures are d-definable.

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[2] Y. L. Ershov, and E. A. Palyutin, ***Mathematical Logic***, FIZMATLIT, Moscow, 2011.

[3] I. I. Pavlyuk and S. V. Sudoplatov, *Families of theories of abelian groups and their closures*. ***Bulletin of Karaganda University. Series "Mathematics"***, vol. 90 (2018), pp. 72–78.

[4] S. V. Sudoplatov, *Ranks for families of theories and their spectra*, 2019, `arXiv:1901.08464v1 [math.LO]`.

[5] W. Szmielew, *Elementary properties of Abelian groups*. ***Fundamenta Mathematicae***, vol. 41 (1955), pp. 203–271.

► IAROSLAV PETIK, *Reading Feyerabend*: *from epistemic anarchism to anarchism in foundations of formal systems*.

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Paul Feyerabend was a famous philosopher of science who developed a theory of scientific anarchism. It claims that rationality and scientific method are only the products of one separate tradition of thought which competes with numerous other traditions [1]. In this schema science is an eclectic set of different competing systems which functions as an evolving system. But there is no general criterion like rationality for the process of selection. On the other hand the question of foundations of formal systems asks the question what is the ontological foundation for any kind of formal system mathematics, logic system, the language of programming, etc. The program of logicism was aimed at proving that all the chapters of mathematics can be reduced to purely logical constructions. Different philosophical theories in mathematics claim that all the mathematics can be reduced to theory of sets, theory of categories, some constructive principles, etc. None of these attempts were eventually successful. The idea of the thesis is to extrapolate and anarchic ideas of Feyerabend on the question about foundations of formal systems. Maybe attempts to find the main formal system were all unsuccessful because there is no such system. There is of course the question of practice in the Feyerabend's conception and its counterpart for the case of formal systems. Probably the role of practice should be admitted in this case as well but in a more specified form. In conclusion it should be said that if the anarchism is eligible for the domain of formal systems than the question about foundations of these systems should be also shifted to the study of cooperation of different systems as equal competing structures.

[1] P. Feyerabend, ***Science in Free Society***, Verso, London, 1982.

► A. V. RAVISHANKAR SARMA, *Belief revision based on abductive reasoning*.

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Belief revison is concerned with the adjustment of currently held beliefs in the light of new information, particularly when the old belief are contradicting the new information [1]. This article discusses the role of abductive reasoning—that is, reasoning in which explanatory hypotheses are formed and evaluated, in the change of beliefs. Recent work in artificial intelligence and Philosophical logic recognizes the importance of abductive reasoning within the process of belief revision, discovery, creativity. The cental idea of the article is that agent seek explanations together with its justification into the agent's current epistemic state before integrating the new information. In the process, an agent given various potential explanations, need to chose the best possible explanation amongst the other competing explanations. We propose an ordering explanations based on the heirarchies of ordering of beliefs called abductive entrenchment ordering of beliefs. This is modification of Pagnucco, Nayak and

Foo's model [3], in two different ways. First, it proposes abductive entrenchment based on causal explanation and second, it takes care of some of the semantic propertoes such as causal properties, causal explanation, causal relevance, with the belief revision process. The presence or lack of these semantic properties leads to the better understanding of ordering of explanations. We also take insights from Kuhn's [2] exhaustive virtues for the theory choice, including accuracy, consistency, scope, simplicity and fruitfulness.

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[2] T. Kuhn, ***Objectivity, Value Judgment, and Theory Choice, in The Essential Tension: Selected Studies in Scientific Tradition and Change***, University of Chicago Press, Chicago and London, 1977.

[3] M. Pagnucco, A. C. Nayak, and N. Y. Foo, *Abductive reasoning, belief expansion and nonmonotonic consequence*, ***Proceedings of the ICLP–95 Joint Workshop on Deductive Databases and Logic Programming and Abduction in Deductive Databases and Knowledge-based Systems***, 1995.

► NOAH SCHOEM, *Destruction of ideal saturation*.
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An ideal I on κ is κ^+-saturated if every antichain of $(P(\kappa)/I, \leq_I)$ has cardinality $\leq\kappa$, and is κ^+-presaturated if I is precipitous and the forcing $(P(\kappa)/I, \leq_I)$ preserves κ^+. We answer an open question of [1] of whether there is a forcing extension that destroys κ^+-saturation of ideals on κ while preserving their κ^+-presaturation in the affirmative.

[1] S. Cox and M. Eskew, *Strongly proper forcing and some problems of Foreman*. ***Transactions of the American Mathematical Society***, vol. 371 (2019), pp. 5039–5068.

► SOURAV TARAFDER, *Foundations of mathematics in a model of paraconsistent set theory*.
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Based on the Boolean-valued model construction of classical set theory, we constructed generalised algebra-valued models in [1]. We defined a three-valued algebra $\mathbf{PS}_3$ such that its logic is paraconsistent [3], and the $\mathbf{PS}_3$-valued model $\mathbf{V}^{(\mathbf{PS}_3)}$ validates the negation-free fragment of ZF [1]. In [2], we studied ordinal numbers in $\mathbf{V}^{(\mathbf{PS}_3)}$.

In this talk, we shall discuss properties of the natural numbers in $\mathbf{V}^{(\mathbf{PS}_3)}$. We consider the ordinal ω (as defined in [2]) as the set of natural numbers and prove that this is an inductive set; from this, we conclude that mathematical induction holds in $\mathbf{V}^{(\mathbf{PS}_3)}$ and discuss the arithmetic of natural numbers in this model. Using the standard definition of sizes of sets via bijective functions, we shall define the notion of cardinality in our model and prove some classical theorems such as Cantor's theorem on the size of the power set of a set.

[1] B. Löwe and S. Tarafder, *Generalized algebra-valued models of set theory*. ***Review of Symbolic Logic***, vol. 8 (2015), no. 1, pp. 192–205.

[2] S. Tarafder, *Ordinals in an algebra-valued model of a paraconsistent set theory*, ***Logic and its Applications, 6th ICLA, vol. 8923,*** (M. Banerjee and S. Krishna, editors), Springer-Verlag, Berlin, Mumbai, India, 2015, pp. 195–206.

[3] S. Tarafder and M. K. Chakraborty, *A paraconsistent logic obtained from an algebra-valued model of set theory*, ***New Directions in Paraconsistent Logic, 5th WCP, vol. 152*** (J.-Y. Béziau, M. K. Chakraborty, and S. Dutta, editors), Springer, New Delhi, Kolkata, India, 2016, pp. 165–183.

► SEBASTIEN VASEY, *Forking and categoricity in nonelementary model theory*.
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The classification theory of elementary classes was started by Michael Morley in the early sixties, when he proved that a countable first-order theory with a single model in some uncountable cardinal has a single model in all uncountable cardinals. The proof of this result, now called Morley's categoricity theorem, led to the development of forking, a notion of independence jointly generalizing linear independence in vector spaces and algebraic independence in fields and now a central pillar of modern model theory.

In recent years, it has become apparent that the theory of forking can also be developed in several nonelementary contexts. Prime among those are the axiomatic frameworks of accessible categories and abstract elementary classes (AECs), encompassing classes of models of any reasonable infinitary logics. A test question to judge progress in this direction is the forty year old eventual categoricity conjecture of Shelah, which says that a version of Morley's categoricity theorem should hold of any AEC. I will survey recent developments, including the connections with category theory and large cardinals, a theory of forking in accessible categories (joint with M. Lieberman and J. Rosický), as well as the resolution of the eventual categoricity conjecture from large cardinals (joint with S. Shelah).

► ANDREAS WEIERMANN, *A unifying approach to Goodstein principles*.
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The Goodstein principle is arguably the most elementary principle which is independent of first order Peano arithmetic. In our presentation we discuss general properties of Goodstein principles which allow to formulate natural variants of the Goodstein principle which do not depend an a specifc notion of base-k representations of natural numbers.

Acknowledgment. This is in part joint work with T. Arai, D. Fernández Duque, and S. Wainer.

[1] T. Arai, D. F. Duque, S. Wainer, and A. Weiermann, *Predicatively unprovable termination of the Ackermannian Goodstein process*, submitted.

[2] E. A. Cichon, *Short proof of two recently discovered independence results using recursion theoretic methods*. ***Proceedings of the American Mathematical Society***, vol. 87 (1983), pp. 704–706.

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[4] ———, *Transfinite ordinals in recursive number theory*. ***The Journal of Symbolic Logic***, vol. 12 (1947), pp. 123–129.

[5] L. Kirby and J. Paris, *Accessible independence results for Peano arithmetic*. ***Bulletin of the London Mathematical Society***, vol. 14 (1982), no. 4, pp. 285–293.

[6] A. Weiermann, *Ackermannian Goodstein principles for first order Peano arithmetic*, ***Sets and Computations***, Lecture Notes Series, Institute for Mathematical Sciences, National University of Singapore, vol. 33, World Scientific Publishing, Hackensack, NJ, 2018, pp. 157–181.

► PEDRO H. ZAMBRANO, *Tameness in classes of generalized metric structures*: *quantale-spaces, fuzzy sets, and sheaves*.
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Tameness is a very important model-theoretic property of abstract classes of structures, under the assumption of which strong categoricity [4, 7] and stability transfer theorems [2, 8] tend to hold. We generalize the argument of Lieberman and Rosický [5]—based on Makkai and Paré's result on the accessibility of powerful images of accessible functors [3] under the existence of a proper class of almost strongly compact cardinalities [1]—that tameness holds in classes of metric structures, noting that the argument works just as well for structures with underlying Q-spaces, Q a reasonable quantale. Dropping the reflexivity assumption from the definition of metrics, we obtain a similar result for classes with underlying partial metric spaces: through straightforward translations from partial metrics to fuzzy sets and sheaves, we obtain, respectively, fuzzy and sheafy analogues of this result.

Acknowledgment. This is joint work with Michael Lieberman and Jiří Rosický.

[1] J. Bagaria and M. Magidor, *Group radicals and strongly compact cardinals*. ***Transactions of the American Mathematical Society***, vol. 366 (2014), pp. 1857–1877.

[2] J. Baldwin, D. Kueker, and M. VanDieren, *Upward stability transfer for tame abstract elementary classes*. ***Notre Dame Journal of Formal Logic***, vol. 47 (2006), no. 2, pp. 291–298.

[3] A. Brooke-Taylor and J. Rosický, *Accessible images revisited*. ***Proceedings of the American Mathematical Society***, vol. 145 (2017), no. 3, pp. 1317–1327.

[4] R. Grossberg and M. VanDieren, *Categoricity from one successor cardinal in tame abstract elementary classes*. ***Journal of Mathematical Logic***, vol. 6 (2006), no. 2, pp. 181–201.

[5] M. Lieberman and J. Rosický, *Hanf numbers via accessible images*. ***Logical Methods in Computer Science***, vol. 13 (2017), no. 2:11, pp. 1–15.

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[7] S. Vasey and S. Shelah, *Categoricity and multidimensional diagrams*, `arXiv:1805.06291`.

[8] P. Zambrano, *A stability transfer theorem in d-tame metric abstract elementary classes*. ***Mathematical Logic Quarterly***, vol. 58 (2012), no. 4–5, pp. 333–341.

Abstracts of articles submitted by title

► JOHN CORCORAN, *Russell 1919 on fractions*.
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We investigate the treatment of fractions in Russell's 1919 classic *Introduction to Mathematical Philosophy* [3]. In contrast to rational numbers, every fraction has an integral numerator and a nonzero integral denominator, but usage varies on exactly which integers are involved (see [4, pages 161ff]). Our interest was first drawn to the topic by the following surprising result—paraphrasing Russell's page 66, as in [1].

Russell 1919's Fraction–Separator Theorem: *the fraction whose numerator is the sum of the numerators of two unequal given fractions and whose denominator is the sum of the denominators is between the two given fractions—excluding cases when the sum of the denominators equals zero.*

Although Russell gives none, proof is obtainable from page 270 of De Morgan 1831 [2].

The paraphrase differs from Russell's words *mainly* in adding the exclusion, which we then took to correct a minor oversight: one denominator can't be the negative of the other. Russell himself had earlier considered negative fractions on page 26: "Here we have first a series of negative fractions with no end, and then a series of positive fractions with no beginning".

His usage on page 26 makes it impossible to determine whether he considered $0/1$ to be a fraction. However, later, on page 84 Russell writes about fractions as though the numerators and denominators were all and only positive integers—as on page 54 of Whitehead 1911 [5].

Russell's [3] is unusually critical of unwarranted "identification" of distinct number classes, for example, page 63 warns against thinking that "a fraction whose denominator is 1 may be identified with the natural number which is its numerator". Nevertheless, it never distinguishes fractions from rationals and, worse, it occasionally confuses fractions with certain ratios and with certain relations.

[1] J. Corcoran, *Russell 1919's fraction-separator theorem*, this Bulletin, vol. 24 (2018), p. 381.

[2] A. De Morgan, ***Study and Difficulties of Mathematics***, Open Court, 1831/1943.

[3] B. Russell, ***Introduction to Mathematical Philosophy***, Dover, 1919.

[4] P. Suppes, ***Axiomatic Set Theory***, Dover, 1960/1972.

[5] A. Whitehead, ***Introduction to Mathematics***, Oxford University Press, 1911.

► JOHN CORCORAN AND MILES RIND, *What syllogisms are: three views, eight centuries*.
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At issue is the nature of "the syllogisms" in *Prior Analytics* [6]. For centuries from the 1200s, the dominant view was enshrined in the medieval mnemonic "Barbara–Celarent" [3]: syllogisms are certain valid premise-conclusion arguments commonly mislabeled "inferences". In the mid-1900s many logicians adopted the Łukasiewicz view [4]: syllogisms are certain true universal propositions informally called "implications". A fruitful two-sided debate ensued producing evidence weakening the Łukasiewicz case.

Toward the last quarter of the 1900s, a third contender appeared. Independently, Corcoran [2] and Smiley [5] took the class of syllogisms to *exclude* propositions while *including* not only the valid arguments recognized as syllogisms in medieval times but also deductions establishing validity—notably the indirect deductions known as "*per-impossibile* syllogisms" [5, p. 136]. A deduction contains, besides an argument's premises and conclusion, reasoning chains showing that the premises' information contains the conclusion's.

The debate became three-sided for some years. Today the Łukasiewicz view lacks defenders leaving the debate to the medieval and Corcoran-Smiley views.

We detail the three views—emphasizing differences. For example, the medieval syllogisms are valid but 'true' does not apply to them, though their premises and conclusions are all true or false. The Łukasiewicz syllogisms are all true but the term 'valid' as applied to arguments is inappropriate. The Corcoran–Smiley syllogisms cannot be said to be true in the sense applicable to propositions but they are all valid in the sense that their conclusions follow from their premises. Moreover, many of them containing reasoning are "cogent" [1] in an epistemic sense inapplicable to Łukasiewicz and medieval syllogisms.

[1] J. Corcoran, *Argumentations and logic*. ***Argumentation***, vol. 3 (1989), pp. 17–43.

[2] ———, *Aristotle's prototype rule-based underlying logic*. ***Logica Universalis***, vol. 2 (2018), pp. 9–35.

[3] J. Corcoran, D. Novotný, and K. Tracy, *Deductions and reductions decoding syllogistic mnemonics*. ***Entelekya Logico-Metaphysical Review***, vol. 2 (2018), pp. 5–39.

[4] J. Łukasiewicz, ***Aristotle's syllogistic***, Oxford University Press, 1951.

[5] T. Smiley, *What is a syllogism*? ***Journal of Philosophical Logic***, vol. 2 (1973), pp. 136–154.

[6] R. Smith, ***Aristotle's Prior Analytics***, Hackett, 1989.

► JOHN CORCORAN AND KEVIN TRACY, *Validity, soundness, truth, known validity, and known truth*.
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Valid arguments are those whose conclusions follow from their premise-sets. Some valid arguments have false conclusions. In such cases a premise is false. No valid argument with all true premises has a false conclusion [1].

Valid arguments with all their premises true are sometimes called *sound*. However, usage varies. Some established logicians use 'sound' as a synonymous substitute for 'valid' [4, 2]; others reserve 'sound' for other uses [1, 5]. Some classic texts don't use 'sound' in any logical senses [6].

Some valid arguments with all premises true have conclusions that are not known to be true. In some cases the premises are not all known to be true. In some cases the arguments are not known to be valid.

In particular, contrary to several logic texts, some "sound" arguments have conclusions that are not known to be true. In some cases the premises of a "sound" argument are not known to be true. In some cases a "sound" argument is not even known to be valid. Moreover, in some cases, no person who knows the truth of the premises knows the validity of the argument. In fact, most "sound" arguments are not known to be "sound".

"Sound" arguments are not proofs. Even arguments known to be "sound" are not proofs [1]. However, every proof contains an argument known to be "sound" by those people who recognize it as a proof, that is, by those for whom it produces knowledge that its conclusion is true [1].

This article quotes and analyses dozens of passages contradicting one or more of the above basic points. Some are in recent publications [3]. Some are in older publications [5].

[1] J. Corcoran, *Argumentations and logic*. ***Argumentation***, vol. 3 (1989), pp. 17–43.

[2] E. J. Lemmon, ***Beginning Logic***, Hackett, 1965/1978.

[3] A. Malpass and M. A. Marfori, ***History of Philosophical and Formal Logic; Aristotle to Tarski***, Bloomsbury, 2017.

[4] B. Mates, ***Elementary Logic***, Oxford University Press, 1972.

[5] W. Quine, ***Philosophy of Logic***, Harvard, 1970/1986.

[6] A. Tarski, ***Introduction to Logic***, Dover, New York, 1995.

▸ B. SH. KULPESHOV AND S. V. SUDOPLATOV, *On P-combinations of ordered structures*.
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In a series of articles [1]–[3] properties for combinations of structures are studied. Here we discuss a P-combination of countably many copies of an $\aleph_0$-categorical structure of pure linear order and present a description of its countable spectrum.

If $\langle M_1, <_1\rangle$ and $\langle M_2, <_2\rangle$ are linear orders then their *linearly ordered disjoint combination* (or *concatenation*), denoted by $M_1 + M_2$, is the linear order $\langle M_1 \cup M_2, <\rangle$, where $a < b$ iff $([a, b \in M_1 \wedge a <_1 b]$ or $[a, b \in M_2 \wedge a <_2 b]$ or $[a \in M_1 \wedge b \in M_2])$.

Let $M_i := \langle M_i; <_{M_i}, \Sigma_i\rangle$ be a linearly ordered structure for each $i \in \omega$. We denote by M' a linearly ordered disjoint P-combination of the structures M_i, $i \in \omega$, in the language $\{<, \Sigma, P_i^1\}_{i\in\omega}$, where $\Sigma = \cup_{i\in\omega}\Sigma_i$, and the universe of the combination is $\bigcup_{i\in\omega} M_i$; $P_i(M') = M_i$ for each $i \in \omega$; either $P_k(M') < P_m(M')$ or $P_m(M') < P_k(M')$ for any $k, m \in \omega$ with $k \neq m$, and there are no coinciding relations (but the order relation) and functions acting in distinct P-predicates.

Theorem 1. *Let M be an $\aleph_0$-categorical structure of pure linear order, M' be a linearly ordered disjoint P-combination of ω copies of M. Then $Th(M')$ has either 2^ω countable models or exactly $(k+2)^m \cdot (k^2+3k+2)^s$ countable models for some nonnegative integers k, m and s with $k \geq 1$.*

Observe that if the structure M, in Theorem 1, is weakly o-minimal and 1-indiscernible, then the value $(k+2)^m \cdot (k^2+3k+2)^s$ is transformed into the value $3^m \cdot 6^s$.

[1] S. V. Sudoplatov, *Combinations of structures*. ***Reports of Irkutsk State University. Series "Mathematics"***, vol. 24 (2018), pp. 82–101.

[2] ———, *Closures and generating sets related to combinations of structures*. ***Reports of Irkutsk State University. Series "Mathematics"***, vol. 16 (2016), pp. 131–144.

[3] ———, *Combinations related to classes of finite and countably categorical structures and their theories*. ***Siberian Electronic Mathematical Reports***, vol. 14 (2017), pp. 135–150.